Abstract

Investment in physical capital at the micro level is infrequent and large, or lumpy. The most common explanation for this is that firms face non-convex physical adjustment costs. The model developed in this paper shows that information costs make investment lumpy at the micro level, even in the absence of non-convex adjustment costs. When collecting and processing information is costly, the firm optimally chooses to do it sporadically and to be inactive most of the time. This behavior results in infrequent and possibly large capital adjustments. The model fits plant-level investment rate moments well, and it also matches some higher order moments of aggregate investment rates.

Keywords: investment dynamics, information costs, inattentiveness, lumpy investment

JEL classification: D21, D83, D92, E22
1 Introduction

A sharp distinction is made in the literature concerning investment adjustments. At the aggregate level, investment adjustments are smooth and gradual, whereas at the micro level they are infrequent and large, or lumpy (Caballero, 1999). That is, investment at the micro level is characterized by long periods of subdued activity interrupted by large adjustments, usually referred to as investment spikes.

To reproduce these behaviors, two different specifications of physical capital adjustment costs have traditionally been considered. While a simple investment model with convex adjustment costs provides a good description of the smooth aggregate behavior, the most common explanation of micro-level lumpy adjustments is that firms face non-convex (e.g. fixed) physical adjustment costs, and so they only invest when their existing stocks of capital differ excessively from an optimal level and are otherwise inactive – an (S,s) rule.

Likewise, an alternative (and to an extent complementary) explanation for such lumpy micro behavior that has been recently put forward – although not yet analyzed – is that information processing and investment planning are costly in terms of required time, effort and expense (Basu and Kimball, 2005, Iacoviello and Pavan, 2007 and Veldkamp, 2011). That is, costs of both acquiring and processing information and of planning could make investment lumpy at the micro level, even in the absence of non-convex adjustment costs. Motivated by this conjecture, this article develops a new model of capital adjustment which draws on recent behavioral models based on the assumption that agents face a cost of updating information, and it tests whether the model is consistent with the aforementioned facts about investment.

The paper draws on two broad literatures, which are reviewed in section 2. The first is the literature on investment in physical capital, and the second is on informational frictions in macroeconomics. After briefly describing the benchmark frictionless investment model (section 3.1) and a model with non-convex adjustment costs (section 3.2), section 3.3 presents the new capital adjustment model based on the Reis (2006b) model of inattentiveness. Section 3.4 shows the micro-level capital adjustment dynamics implied by these models, and section 3.5 tests their predictions against U.S. plant-level investment rate data. Section 4 aggregates the behavior of many inattentive firms and assesses whether the inattentiveness model is able to match the aggregate data. Section 5 concludes. Technical details are relegated to appendices A and B.
2 Motivation

This section briefly reviews the two literatures on which this work builds – one on adjustment costs and investment dynamics, and one on inattentiveness in macroeconomics.

Adjustment costs and investment dynamics

Ever since the pioneer analysis of Eisner and Strotz (1963), the workhorse model of the investment literature has been, partly for analytical tractability, a neoclassical model with strictly convex – most often quadratic – costs of adjustment. This model provided a theoretical micro-foundation to justify the inclusion of lagged dependent variables in empirical models of (otherwise static) factor demand (the flexible accelerator model of Clark, 1944, or the flexible user-cost model of Hall and Jorgenson, 1967). Convex costs of adjusting capital induce firms to spread their investment over time, since a series of small adjustments is cheaper than a single large adjustment. Despite the relative success in reproducing the smooth adjustment of investment observed at the aggregate level, there is mounting empirical evidence – summarized in table 1 – showing that capital adjustment at the micro level is lumpy rather than smooth. In many different countries, investment at the micro level is characterized by periods of inactivity and few large adjustments in capital stocks (spikes), which nevertheless account for the bulk of investment.

This very different picture of investment adjustment has led economists to question even more forcefully the convex adjustment costs assumption, since it suggests that the predominant adjustment frictions at the micro level are likely to be non-convex, rather than convex, in nature. Scarf (1960) shows that nonlinear microeconomic adjustments can arise when firms face non-convex adjustment technologies. In Scarf’s model, the adjustment cost is a fixed cost incurred whenever a firm wants to adjust its stock of inventories. To avoid the payment of such a lump-sum cost, the firm invests only when its stock of capital deviates from a target level by more than a certain threshold amount. The optimal adjustment policy in the presence of

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1 See Chirinko (1993) and Bond and Van Reenen (2007) for excellent surveys of traditional investment models, and Hamermesh and Pfann (1996) for a more general survey on factor adjustment costs.
2 Further evidence for the U.S. comes from Caballero et al. (1995), Doms and Dunne (1998) and Cooper et al. (1999) (LRD database), and Becker et al. (2006) (Census Bureau’s Annual Survey of Manufactures data). For Italy see Del Boca et al. (2008); for Sweden see Carlsson and Laseen (2005); for the Netherlands see Letterie and Pfann (2007); for Hungary see Reiff (2010).
fixed adjustment costs thus implies periods of inaction interrupted by infrequent episodes of large capital adjustments.

Beginning with Scarf, researchers have studied richer forms of adjustment cost structures to achieve greater consistency with the lumpy micro evidence. Despite substantial improvements in recent years in analytical techniques which can handle these types of nonlinear adjustments, it has been difficult to produce a model that is able to reproduce both the investment spikes and the periods of inactivity of the frequency found in the data. After numerous attempts and refinements, only a few recent models – Cooper and Haltiwanger (2006), Khan and Thomas (2008) and Bachmann et al. (2013) – have been successful in matching the moments from the cross-sectional distribution of plant-level investment rates. Cooper and Haltiwanger (2006) estimate a model that nests alternative specifications of adjustment costs. They find that a model which combines both convex and non-convex adjustment costs (as well as irreversible investment) fits the data reasonably well. Khan and Thomas (2008) instead solve the problem of reproducing observations of both spikes and inaction by assuming that plants may undertake low levels of investment without incurring any adjustment costs. Finally, Bachmann et al. (2013) match plant-level investment rate moments by introducing capital sales and measurement error into their state-dependent model.

Inattentiveness as an alternative micro-foundation for lumpy investment adjustments

Within the literature on informational frictions in macroeconomics, a strand called inattentiveness has emerged recently. The inattentiveness literature, pioneered by Reis (2006b), interprets the concept of “menu cost” introduced by Mankiw (1985) as a cost of acquiring, absorbing and processing information and making decisions and plans based on that information. This cost can be interpreted as the cost in money and time of obtaining and assimilating information, or it could stand for the opportunity cost of taking time to think about and to compute optimal plans. Thus, in what follows I will refer to this cost as information cost.\(^3\)

\(^3\) Some recent lumpy investment models with a single component of adjustment costs include Caballero and Engel (1999), Thomas (2002) and Veracierto (2002). Bertola and Caballero (1990), Abel and Eberly (1994), Cooper et al. (1999), Le and Jones (2005), Cooper and Haltiwanger (2006) and Bloom et al. (2007), among others, consider combinations of convex and non-convex adjustment costs and/or irreversible investment.

\(^4\) A related strand of the literature, introduced by Sims (2003) and known as rational inattention, analyzes incomplete adjustments that arise because of the limited ability of agents to absorb information. In particular, agents update their information and plans frequently (every period) but incompletely. With inattentiveness instead, information and plans are updated infrequently but completely. See Veldkamp (2011) for a comprehensive analysis of informational frictions in macroeconomics and finance.
Table 1: Summary statistics, micro-level data

<table>
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<th>PANEL A: United States</th>
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<th>ASM</th>
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<td>83-96&lt;sup&gt;c&lt;/sup&gt;</td>
<td>60-87&lt;sup&gt;d&lt;/sup&gt;</td>
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<td>84.5</td>
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<td>0.058</td>
<td>0.2057</td>
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<td>1.07</td>
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<td>76-90&lt;sup&gt;f&lt;/sup&gt;</td>
<td>78-91&lt;sup&gt;g&lt;/sup&gt;</td>
<td>91-02&lt;sup&gt;h&lt;/sup&gt;</td>
<td>73-98&lt;sup&gt;i&lt;/sup&gt;</td>
</tr>
<tr>
<td>inaction (%)</td>
<td>1.2</td>
<td>14.7</td>
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<td>84-94&lt;sup&gt;i&lt;/sup&gt;</td>
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<td>30.4 (eq)</td>
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<td>1.9 (la), 4.3 (eq)</td>
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<td>10.1 (la), 63.9 (eq)</td>
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<td>1.2 (la), 11.1 (eq)</td>
<td>4.5 (la), 21.1 (eq)</td>
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<td>4.5 (la), 21.1 (eq)</td>
<td>4.5 (la), 21.1 (eq)</td>
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<td>Sample period</td>
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<td>0</td>
<td>2</td>
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Note. An empty cell means that the value is not reported in the respective paper. When not specified, inaction: $|i/k| < 1\%$; negative investment: $i/k \leq -1\%$; positive investment: $i/k \geq 1\%$; positive spike: $i/k > 20\%$; negative spike: $i/k < -20\%$; serial correlation: $\text{corr}\left[(i/k)_t, (i/k)_{t-1}\right]$.  

† Cooper and Haltiwanger (2006, table 1). LRD: Longitudinal Research Database.  

‡ Gourio and Kashyap (2007a, table 1, capital weights). AMS: Census Bureau’s Annual Survey of Manufactures.  

§ Bayraktar (2002, table 1). Inaction: $|i/k| < 2.5\%$; negative investment: $i/k < 0\%$.  

‖ Bayraktar and Sakellaris (2006, table 2). Inaction: $|i/k| = 0$; negative (positive) investment: $i/k < (>) 0\%$. Positive spike: $i/k > 0.20 + \delta$, where $\delta$ is the depreciation rate.  


‖‖ Nilsen and Schiantarelli (2003, table 1, equipment+buildings). CBS: Central Bureau of Statistic of Norway. Inaction: $|i/k| = 0$; negative (positive) investment: $i/k < (>) 0\%$.  


‖‖‖‖ Bachmann and Bayer (2012, table 21, all firms). Negative investment: $-20\% < i/k < -1\%$; positive investment: $1\% < i/k < 20\%$.  

‖‖‖‖‖ My own calculations based on the ETLA’s database (500 largest Finnish firms, gross investment).  

‖‖‖‖‖‖ Gourio and Kashyap (2007a, table 1, capital weights). NSI: National Statistics Institute of Chile. Inaction: $|i/k| < 2\%$; positive investment: $|i/k| > 2\%$.  

‖‖‖‖‖‖‖ Fuentes et al. (2006, table 1). n.a.: not available. Inaction: $|i/k| = 0$; negative (positive) investment: $i/k < (>) 0\%$.  

‖‖‖‖‖‖‖‖ INEGI: National Institute of Statistics, Geography, and Information. AMS: Annual Manufacturing Surveys. Inaction: $|i/k| = 0$; negative (positive) investment: $i/k < (>) 0\%$. (la): land; (eq): equipment (machinery+transport+other).  

‖‖‖‖‖‖‖‖‖ Contreras (2008, table 1).  

‖‖‖‖‖‖‖‖‖‖ Gebreyesus (2009, table 2, total fixed assets). CSA: Ethiopian Central Statistics Authority. Inaction: $|i/k| = 0$; negative (positive) investment: $i/k < (>) 0\%$.  

‖‖‖‖‖‖‖‖‖‖‖ My own calculations based on data available from Bigsten et al. (2005, tables 1-3). RPED: Regional Programme on Enterprise Development. Negative spike: $i/k < -10\%$.  


Inattentiveness is the optimal response to such information cost since agents rationally choose to update their information sets and to make new plans only sporadically at optimally chosen dates, and to be inattentive to new information in between observation dates. As a consequence, expectations conditional on old information continue to influence current decisions, so agents only react with a delay to news/shocks.

The inattentiveness literature is expanding rapidly and this approach has been successfully used to explain the behavior of price-setting firms (Mankiw and Reis, 2002, Reis, 2006b, Alvarez et al., 2011 and Bonomo et al., 2011), workers (Mankiw and Reis, 2003), consumers (Reis, 2006a) and investors in financial markets (Gabaix and Laibson, 2002, Abel et al., 2007, 2012 and Alvarez et al., 2012). However, so far, the role of information costs in shaping investment in physical capital by firms has not been analyzed. This paper tries to fill this gap.

In the real world, firms do not adjust capital immediately with every change in demand conditions or productivity, even if they possess all the information. A plausible reason for this is that enormous resources (in money and time) are devoted to activities such as managerial decision making and evaluating potential investments, which involves processing and synthesizing the relevant information – and no manager can truly decide everything in a timely fashion.

Accordingly, the key assumption in this model (which also represents the only departure from an otherwise frictionless model) is that the firm faces a cost of gathering and processing information, instead of a physical cost of adjusting its stock of capital. This cost induces the firm to make infrequent investment decisions. In between observation dates, the firm is inattentive and undertakes continuous investment (to compensate for depreciation), but when the firm does update its information, it reacts to all the information since its last observation date, and the stock of capital instantaneously jumps (since there are no adjustment costs) to the optimally chosen level. As a result, it is likely to have investment spikes at those observation dates, especially if the firm stays inattentive for long periods. The dynamics of investment under inattentiveness therefore implies lumpy adjustments: periods of low activity are interrupted by possibly large capital adjustments.

Thus, inattentiveness – due to a cost of acquiring and processing information – provides an alternative

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5 The aforementioned papers analyze the consequences of inattentiveness in partial equilibrium frameworks. Examples of general equilibrium models with inattentiveness are Ball et al. (2005), Mankiw and Reis (2006, 2007), Reis (2009a,b) and Verona (2011).
micro-foundation for the lumpiness of capital adjustment observed at the micro level.

The infrequent investment planning underlying the inattentiveness model also matches well with anecdotal evidence on the frequency of decision making in firms (Bloom, 2009). In the real world, firms have management meetings on a regular basis – within each firm, a variety of managers (CEO, CFO, and plant managers) meet to discuss and decide on large investment projects, probably once or twice a year. These meetings are often either scheduled at fixed periodicity (e.g. the first Monday of each quarter/semester) or set up for a future date agreed at the end of each meeting. As I will show later, the inattentiveness model predicts that the firm decides the date of the next meeting during the current meeting, and the period of inattentiveness is fixed.\(^6\) Thus, the inattentiveness model captures some of the descriptions of how firms make decisions in the real world.

3 A tale of dynamic models of micro-level capital adjustment

I begin this section by describing a neoclassical investment model without capital adjustment costs of any kind. This model serves as a reference for building more realistic investment models. Next, I describe a model with non-convex (fixed) adjustment costs. Then, I introduce the model with information costs and derive some theoretical results describing the firm’s capital adjustment behavior. Finally, I describe and contrast the capital adjustment dynamics at the micro level implied by these models, and test their ability to match plant-level data.

3.1 A frictionless neoclassical investment model

Here I describe a simplified version of the investment model presented in Abel and Eberly (2011). Let \(\Pi_t = Z_t^{1-\alpha} K_t^\alpha\) be a reduced form revenue function obtained from the firm’s optimization over freely adjustable factors of production, where \(K_t\) denotes the current capital stock, \(Z_t\) the current period profitability shock and \(0 < \alpha < 1\) the degree of monopoly power.\(^7\)

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\(^6\) This last result may be easily modified so that the period of inaction may depend, for instance, on business cycle conditions.

\(^7\) Let \(P_t = h_t Y_t^{\varepsilon - \frac{1}{\varepsilon}}\) be the demand curve where \(h_t > 0\) is a demand shock and \(\varepsilon > 1\) is the price elasticity of demand. Let \(Y_t = A_t \left( K_t^\gamma N_t^{1-\gamma} \right)^s\) be the production function where \(\gamma\) is the capital share in a Cobb-Douglas production function and \(s\) the
The firm can purchase or sell capital instantly and without any adjustment costs, at a constant price normalized to one. Hence the Jorgensonian user cost of capital equals the discount rate of the firm, \( r \), and the depreciation rate, \( \delta \).\(^8\) Operating profits (revenue minus the user cost of physical capital) are therefore given by

\[
\Pi_t = Z_t^{1-\alpha} K_t^\alpha - (r + \delta) K_t.
\]

(1)

Since there are no capital adjustment costs and investment is completely reversible, the optimal capital stock at each point in time is determined by maximizing static operating profits in equation (1) with respect to \( K_t \). The first order condition yields the firm’s optimal stock of capital:

\[
K_t^* = M^{\frac{\alpha}{\alpha-1}} Z_t,
\]

(2)

where \( M \equiv \left(\frac{r+\delta}{\alpha}\right)^{-\frac{\alpha}{\alpha-1}} \). Throughout this paper, I will refer to \( K_t^* \) as the static optimum. According to the model, the static optimum is as volatile as the profitability shock, and the response of the capital stock mimics that of the shock. These two predictions are at odds with the conventional wisdom that the capital stock adjusts either continuously and in small amounts (macro level), or infrequently and in large amounts (micro level).

### 3.2 A model with non-convex adjustment costs

Given the mismatch between the frictionless model’s predictions and the data, researchers have introduced different forms of adjustment costs. Beyond other costs associated with the purchase of capital (i.e. the user cost), the very act of adjusting the capital stock entails real costs. Ideally, the optimizing firm would choose a stochastic process for \( K_t \), such that \( K_t = K_t^* \ \forall t \). However, if altering its stock of capital is costly, the firm has to trade off the benefits of tracking \( K_t^* \) more closely against the costs of doing so.

degree of returns to scale. Maximization of profit over the flexible labor factor, \( N_t \), leads to a reduced form revenue function, \( \tilde{\Pi}_t = Z_t^{1-\alpha} K_t^\alpha \), where \( Z_t \) reflects productivity, the demand for the firm’s output and/or the wage rate as well as structural parameters. The exponent on capital is \( \alpha \equiv \gamma s \left(1 - \frac{1}{\varepsilon}\right) \left[ 1 - (1 - \gamma) s \left(1 - \frac{1}{\varepsilon}\right) \right] \). See Abel and Eberly (2011) for further details.

\(^8\) Jorgenson (1963) shows that, if the level of capital inputs can be freely adjusted, the firm should invest until the marginal profit from an extra unit of capital is equal to the user cost of capital. The user cost represents the opportunity cost of holding a unit of capital for one period. It is usually defined as the sum of three terms: the firm’s required rate of return, the depreciation rate and the expected rate of change in the price of capital goods. Here the latter term is zero since, by assumption, the price of capital is constant.
As first shown by Scarf (1960), lumpy behavior arises naturally when adjusting the stock of capital entails a fixed cost. Here I briefly describe a simple model of this type, and refer the reader to e.g. Caballero (1999) for an in-depth analysis of models with adjustment costs.

Let $x$ denote the current state and $x'$ the state at the next adjustment date $(T)$, and let the adjustment cost incurred in changing the capital stock at any given time $T$ be proportional to the capital stock (before adjustment): $\Phi K_T$. At every moment, the firm considers whether the disparity between the current capital stock and the static optimum is sufficiently large to justify paying the adjustment cost and reoptimizing over the capital. At any normalized time 0, the Bellman equation is

$$V(x) = \max_{K_t,T} E \left\{ \int_0^T e^{-rt} \left[ Z_t K_t^{\alpha} - (r + \delta) K_t \right] dt + e^{-rT} \left[ -\Phi K_T + V(x') \right] \right\},$$

where $T$ is the first stopping time.

I describe the adjustment policy in terms of two trigger levels, $L$ and $U$, where $L < U$. Let $\kappa_T$ be the deviation between the static optimum and the current capital before adjustment takes place: $\kappa_T = K_T^* - K_T$. If $\kappa_T$ is smaller (larger) than or equal to a trigger level $L$ ($U$), I assume that the firm increases (decreases) its capital stock such that $\kappa_T = 0$ after adjustment. There is therefore an investment gulp each time $\kappa_T$ reaches these trigger levels. On the other hand, if the value of $\kappa_T$ is between trigger levels, I assume that the firm undertakes maintenance investment to compensate for depreciation without incurring any adjustment costs. Under these assumptions, the optimal policy is described by the following rule:

$$K_t^{Ss} = \begin{cases} K_t^* & \text{if } \kappa_T \geq U \text{ or } \kappa_T \leq L \\ K_{t-1} & \text{if } L < \kappa_T < U \end{cases},$$

The trigger levels as well as the optimal dynamic target $\kappa_T$ depend on parameters like the size of the adjustment costs $\Phi$ or the volatility of the profitability shock $Z_t$ (see e.g. Grossman and Laroque, 1990 for further details). Moreover, as Caballero (1999) points out, the optimal dynamic target may be different from the static one.

To model the behavior of firms when inactive (i.e. when the capital stock is between the trigger levels), the literature has followed three different approaches. One is to let capital be eroded by depreciation, as in Bertola and Caballero (1990), Caballero (1993), Thomas (2002), Khan and Thomas (2003) and Gourio and Kashyap (2007b). The second one (Bachmann et al., 2013) assumes that every period firms have to partially replace parts and machines so that they do not depreciate completely, otherwise production cannot continue. The third approach (see e.g. Sveen and Weinke, 2007 and Khan and Thomas, 2008) assumes that small capital adjustments are exempt from adjustment costs. Here I follow the latter approach and assume that firms fully compensate depreciation every period.
and investment is given by

\[ I_t = \begin{cases} 
K_t^* - (1 - \delta) K_{t-1} & \text{if } \kappa_{T-} \geq U \text{ or } \kappa_{T-} \leq L \\
\delta K_{t-1} & \text{if } L < \kappa_{T-} < U 
\end{cases} \]

Note that, in (3) the optimal stopping time for adjustment, \( T \), depends on the evolution of the state variable \( Z_t \). Adjustment is therefore state-dependent: the firm observes the state of the economy each instant and accordingly decides whether it is optimal to adjust its stock of capital or to stay inactive. Once the decision to act is made, adjustment is instantaneous and capital jumps to the optimally chosen level. The optimal adjustment policy in the presence of fixed adjustment costs thus implies periods of low activity followed by large capital adjustments.

### 3.3 The model with information costs

In this subsection I present an alternative model of capital adjustment based on the inattentiveness model of Reis (2006b). The key assumption in this model is that information gathering and processing is costly – rather than adjusting the stock of capital, as is common in the investment literature. Compared to the model described in section 3.1, the firm here faces one additional constraint: it incurs a cost of updating its information sets. When the firm decides to incur such a cost, conditional on the information obtained, it makes two decisions.

The first decision is when to update the information again. The optimal length of inattentiveness trades off the costs incurred by acquiring information against the gains (losses) from having a re-optimized (outdated) plan. While on the one hand, being inattentive saves on the costs of collecting information, on the other hand, it implies that decisions far in the future are made with outdated information. At some point the costs of following an outdated plan will exceed the costs of updating information, so it becomes optimal for the firm to update the information and to make a new investment plan.

The second decision is the plan for capital adjustments, which gives the path for the amount of physical capital to use until the next observation date. In between observation dates, the dynamics of capital follows
this pre-determined plan, regardless of news on the economy. Thus, when acting inattentively, the firm undertakes continuous investment to compensate for depreciation so as to keep its capital stock in line with the optimal plan. The model thus captures the fact that firms engage continuously in some investment (e.g. maintenance), as Doms and Dunne (1998) report for the U.S. economy. At observation dates, in contrast, information is revealed and the stock of capital jumps to its static optimum (since investment is frictionless and completely reversible). Thus we are likely to observe large adjustments at those observation dates.

So, having informally described the dynamics of investment under inattentiveness, I now introduce the formal problem and derive the optimality conditions describing the capital adjustment behavior.

3.3.1 The inattentive firm’s problem

Time is continuous and infinite. Let $x_t$ be the state vector, which is generated by a continuous time stochastic process defined on a standard filtered probability space with filtration $F = \{F_t, t \geq 0\}$. I assume that $x_t$ is a first-order Markov process. The state at a given date $t + \tau$ is then a function of $x_t$ and a set of innovations $u^\tau = (u_t, u_{t+\tau}]$, so that $x_{t+\tau} = \Psi (x_t, u^\tau)$ gives the transition between the state at date $t$ and the state at date $t + \tau$, which is assumed to be differentiable.

The observation (or planning) dates are denoted by $D(i)$, where $i \in \mathbb{N}_0$ and $D(i+1) \geq D(i)$ for all $i$ with $D(0) \equiv 0$. The periods of inattentiveness are defined recursively as $d(i) = D(i+1) - D(i)$. The firm’s optimal choice of planning dates defines a new filtration $\Im = \{\Im_t, t \geq 0\}$ such that $\Im_t = F_{D(i)}$ for $t \in [D(i), D(i+1))$.

Whenever the firm decides to update its information and plans, it incurs a non-negative finite cost given by $\theta_t \equiv \theta(x_t)$. It then chooses when to collect the information again as well as a plan for capital adjustments up to the next observation date, $K(t) = K[D(i), D(i+1))$. Two remarks are worthwhile. First, note that the firm can choose the next planning date either at the current planning date or at any future date. However, since it does not receive any new information while inattentive, its choice will be the same irrespective of when it is made. Second, the firm’s choices at any time $t$ must be measurable with respect to $\Im_t$. That is, the firm’s capital choice for time $t$ is conditional on the information it has at time $t$, which by assumption coincides with the information available at the last planning date.
The firm maximizes the expected value of operating profits, net of information costs. The firm’s problem can be written as

\[
\max_{\{K(i), D(i)\}_{i=0}^\infty} E \left\{ \sum_{i=0}^{\infty} \left[ e^{-rt} \Pi_i dt - e^{-rD(i+1)} \theta \left( x_{D(i+1)} \right) \right] \right\}
\]

\[\text{(4)}\]

\[\text{s.t. \quad \{D(i), K(i)\} are } \mathcal{F}\text{-adapted} \]

\[\text{(5)}\]

\[x_{t+\tau} = \Psi \left( x_t, u^\tau \right) \]

\[\Pi_t = Z_t^{1-\alpha} K_t^\alpha - (r + \delta) K_t. \]

\[\text{(6) \quad \text{(7)}\]

Note that if the costs of acquiring information are always 0, the firm optimally chooses to always be attentive.

The problem (4)-(7) has a recursive structure between observation dates. Let \( x \) denote the state at the current observation date and \( x' \) the state at the next observation date. I can then rewrite the problem as

\[V(x) = \max_{\{K_t\}_{t=0}^d} \left\{ \int_0^d e^{-rt} \left[ Z_t^{1-\alpha} K_t^\alpha - (r + \delta) K_t \right] dt + e^{-rd} E \left[ -\theta \left( x' \right) + V \left( x' \right) \right], \text{ s.t. } x' = \Psi \left( x, u^d \right) \right\}, \]

\[\text{(8)}\]

where the measurability constraint (5) is imposed by having passed the expectation operator through \( \{K, d\} \), so that these choices are made conditional on the information available at the current observation date. The solution to the problem in (8) is a pair of functions, \( K(x, t) \) and \( d(x) \), determining the optimal plan for capital from time 0 to time \( d \) and the next observation date \( d \).

At a first sight, the problem in (8) seems similar to the optimal stopping problem in (3). However, recall from section 3.2 that adjustment in (3) – i.e. the choice of \( T \) – is state-contingent. In the inattentiveness model by contrast, in between adjustments, the firm rationally chooses to be inattentive and not to plan. The inattentive firm adjusts at optimally chosen dates regardless of the state of the economy at those dates. Adjustment with inattentiveness – i.e. the choice of \( d^* \) – is therefore recursively time-contingent and independent of the current state, but is a function of the state at the past observation date.
3.3.2 The optimality conditions

The first order condition of (8) with respect to \( d \) is

\[
\Pi(x, d) = E \left\{ r \left[ V(x') - \theta(x') \right] + \left[ \theta_x(x') - V_x(x') \right] \frac{\partial \Psi(x, u^d)}{\partial d} \right\}.
\] (9)

By weighting the benefits of adjusting against the costs incurred by collecting information, equation (9) implicitly defines the optimal length of inattentiveness. On the left-hand side is the value from extending the inattentiveness interval, which equals the profits from sticking with the outdated plan for capital. The right hand side captures the value of updating information at time \( d \), which is the sum of two terms. The first term is the present value of having obtained new information and re-planned, which is the difference between the value of having a new plan and the cost of making it. The second term is the cost of updating information at date \( d \) rather than at another instant in which the costs and benefits of a fresh plan may change.

Differentiating equation (8) with respect to \( K_t \) and setting the derivative equal to zero yields the optimal plan for capital:

\[
K_{IN}(x, t) = M^{1/\alpha} \left[ E \left( Z_t^{1-\alpha} \right) \right]^{1-\alpha}.
\] (10)

Finally, the envelope conditions with respect to each component \( j \) of the state vector \( x \) are

\[
V_j(x) = \int_0^d e^{-rt} \Pi_j(x, t) \, dt + e^{-rd} E \left[ \left( -\theta_x(x') + V_x(x') \right) \Psi_j(x, u^d) \right].
\] (11)

Equations (8)-(11) characterize the value function \( V(x) \), the plan for capital \( K_{IN}(x, t) \) and the optimal inattentiveness interval \( d(x) \).

3.3.3 The model’s predictions

I now derive some theoretical implications of the model by making assumptions that lead to a closed-form solution. I assume that the profitability shock, \( Z_t \), follows a trendless geometric Brownian motion with constant volatility \( \sigma > 0 \), so that \( dZ_t = \sigma Z_t \, dz \), where \( dz \) is the increment of a standard Wiener process. I
also assume that the information costs are a fixed share $\Theta$ of profits at observation date.\footnote{Appendix B contains the proofs of all the propositions.}

*Prediction #1: the length of inattentiveness*

Following Reis (2006b), I compute a perturbation approximation of the optimal length of inattentiveness around the point where information is costless to acquire.\footnote{Results are sensitive to the point around which the approximation is taken. For instance, Jinnai (2007) solves the Reis (2006b) model by approximating around the point where firms have asymmetric information. Overall, Jinnai’s approximation predicts shorter inattentiveness intervals than Reis’ approximation.} In this case,

**Proposition 1.** An approximation of the optimal inattentiveness interval is given by

$$d^* = \sqrt{\frac{4\Theta}{\alpha \sigma^2}}.$$

This result gives us the three determinants of inattentiveness. First, $\frac{\partial d^*}{\partial \Theta} > 0$: the higher the cost of collecting information, the longer the inattentiveness. Second, $\frac{\partial d^*}{\partial \alpha} < 0$: the greater the sensitivity of the profit function, the higher the cost of a delayed reaction to news; hence the firm updates its information more frequently. Third, $\frac{\partial d^*}{\partial \sigma} < 0$: the inattentiveness period becomes the shorter, the greater the volatility, since it is costly for the firm to be inattentive in periods of high volatility (which may be considered as periods of great uncertainty).

The last prediction shows that uncertainty increases the responsiveness of a firm, but also its investment activity. That is, in periods of high uncertainty the firm is more attentive, but it also tends to invest more.\footnote{Even though there is no direct empirical evidence on the allocation of attention over the business cycle, it seems intuitive that people – households, firms and policymakers – are more attentive in periods of high uncertainty.}

The latter effect (which arises because there are no adjustment costs, so that the firm in the model always invests at observation dates) is the opposite of the “cautionary effect” of uncertainty on investment found in the literature (Bloom 2009). Future empirical work can test these opposing effects.\footnote{To reduce investment activity at observation dates, I should introduce non-convex adjustment costs so that the firm would only invest if its stock of capital is far from the static optimum. This would require modifying the theoretical structure of the model, as well as the analytical method to solve it. I leave this for future work.}

*Prediction #2: capital adjustment dynamics with inattentiveness*

If $K^*_{D(i)} \equiv M^{\frac{1}{2}} Z_{D(i)}$ denotes the capital chosen by an attentive firm at date $D(i)$ (recall equation 2), then
the optimal capital of an inattentive firm at a observation date $D(i)$ is

$$K_{D(i)}^{IN} = K_{D(i)}^* = M^{\frac{1}{2}} Z_{D(i)}.$$

In between observation dates, the firm is inattentive and

**Proposition 2.** The optimal plan for capital between observation dates, for $D(i) < t < D(i+1)$, obeys the equation

$$K_{t}^{IN} = K_{D(i)}^* e^{-\frac{\alpha^2}{2} \left\{ \left( t - D(i) \right) \right\}}.$$

The optimal capital adjustment policy with inattentiveness is thus fully described by the following equations

$$K_{t}^{IN} = \begin{cases} 
K_{D(i)}^* = M^{\frac{1}{2}} Z_{D(i)} & \forall i \in \mathbb{N}_0 \\
K_{D(i)}^* e^{-\frac{\alpha^2}{2} \left( t - D(i) \right)} & \forall D(i) < t < D(i+1)
\end{cases}.$$ 

**Prediction #3: the dynamics of investment**

If the firm enters period $D(i)$ with capital stock $K^{D(i)}_-$, its capital stock jumps instantly to $K^{D(i)}_+ = K^{D(i)}_- + I_{D(i)}$, where the superscripts “+” and “−” on $K^{D(i)}_-$ denote, respectively, the stock of capital immediately after and immediately before $D(i)$, and $I_{D(i)}$ is the investment gulp at date $D(i)$. Between observation dates, the firm is inattentive and, in order to keep its capital stock in line with the optimal plan, it undertakes continuous investment, $I_{t}^{M}$, to compensate for depreciation. Therefore optimal investment by the inattentive firm is:

$$\begin{cases} 
I_{D(i+1)} = K_{D(i+1)}^+ - K_{D(i+1)}^- = K_{D(i+1)}^* - K_{D(i+1)}^* e^{-\frac{\alpha^2}{2} d^*} & \forall i \in \mathbb{N}_0 \\
I_{t+1}^{M} = \frac{dK_{t}^{IN}}{dt} + \delta K_{t}^{IN} & \forall D(i) < t < D(i+1)
\end{cases}.$$ 

The dynamics of capital adjustment with inattentiveness thus implies periods of relatively low activity fol-
lowed by sporadic episodes of possibly large capital adjustments.

### 3.4 Micro-level capital adjustment dynamics

Having presented the models, I now compare the capital adjustment dynamics implied by different adjustment policies. The upper graph in figure 1 presents sample paths for the static optimum (circle line) and the capital adjustment behavior implied by the model with fixed adjustment costs (square line) and fixed information costs (diamond line). As one can see, both models with fixed costs predict lumpy adjustments, as periods of low investment activity (in which the capital is constant or decreases slightly) are interrupted by possibly large (both positive and negative) capital adjustments. However, their dynamics differ in two important aspects, the size and timing of adjustments. In the model with non-convex adjustment costs, adjustments are always large because the firm only adjusts when its current capital stock is far from the optimum. In the inattentiveness model, instead, the firm expects to be far from the optimum, but it may or may not be, depending on current and past news, so that adjustments may be large or small. For instance, there is a large adjustment around time 650, and a small one at time $950$.

The lower graph in figure 1 helps us understand the difference in the timing of adjustment, i.e. when the firm adjusts. It presents sample paths for the deviation between static optimum and current capital stock for both models with fixed costs. In the model with adjustment costs, the firm observes the state of the economy every instant and adjustments occur at dates $T_1$ and $T_2$, when the departure from the static optimum becomes too large in absolute value, reaching the upper level $U$ or the lower level $L$. In the inattentiveness model, full adjustments occur at fixed intervals $d^*$, regardless of the state at those dates. That is, whenever there is a cost of gathering information, and no direct costs of adjusting capital, the optimal adjustment is time- as opposed to state-dependent.

This example shows that a slightly different interpretation of menu cost – cost of adjusting versus cost of collecting information – leads to different capital and investment dynamics. In particular, shocks have long-lasting real effects if firms make investment decisions according to a time-dependent rule (since only a few firms are attentive and thus aware of the shocks/news), whereas if firms adjust their capital stocks according

---

15 Recall that capital is constant because of the assumption of 100% maintenance investment to fully compensate depreciation.
Figure 1: Examples of capital adjustment dynamics (baseline parameters, see table 2)
to a state-dependent rule, then shocks may have little real effect.

### 3.5 How well do the models fit plant-level data on investment?

This subsection evaluates the ability of the models to match U.S. plant-level investment rate moments. The models are calibrated to annual data, since the plant-level evidence is based on annual surveys. My preferred choices are $r = 0.04$, which is the value typically used in the literature for the discount factor; the depreciation rate $\delta = 0.13$, to match the average investment rate at the plant-level; the degree of monopoly power $\alpha = 0.8$, which lies between the value used by Khan and Thomas (2008) and the value estimated by Cooper and Haltiwanger (2006); and the standard deviation of the profitability shock $\sigma = 0.12$. For the costs of collecting information in the inattentiveness model, I choose $\Theta = 0.020$, which is in line with the evidence provided in Zbaracki et al. (2004).\(^\text{16}\) This parametrization implies that firms remain inattentive for about 5 quarters.\(^\text{17}\) With regard to the model with fixed adjustment costs, I pick the threshold values $U = L = 0.15$, that is, there is no adjustment as long as the current capital stock (before adjustment) is within a $\pm15\%$ band around the static optimum.

Using this parametrization, I generate simulated capital and investment data for a panel of 2500 plants over 17 years.\(^\text{18}\) As a technical matter, instead of generating one normally distributed value per year for the profitability shock $Z$, I divide each year into 240 intervals (days) and generate a normally distributed shock for each interval. Using a finer time grid allows me to choose more precise values for observation dates (recall that observation dates in the inattentiveness model are chosen from a continuous set). Investment rates are computed as total annual investment – the sum of spikes and continuous investment in any given year – divided by the capital stock in the final day of the year.

\(^\text{16}\) Zbaracki et al. (2004) find that the managerial costs (information gathering, decision-making, and communication costs) of price adjustments are about 4.6% (2.8%) of the firm’s net margin (total costs). There is no direct evidence on the actual magnitude of the managerial costs of investment adjustments, but these costs are likely to be significant for investment decisions as well, especially for large investment projects.

\(^\text{17}\) Extensive sensitivity analysis confirmed that the properties of the inattentiveness model are not sensitive to variation in the parametrization. I have considered several empirically plausible values for the parameters ($0.5 < \alpha < 0.9$, $0.010 < \Theta < 0.030$, $0.08 < \sigma < 0.16$ and $0.12 < \delta < 0.14$; $r$ does not affect the dynamics) and I have reported here the calibration that provides the best fit of the model to the micro data.

\(^\text{18}\) This is the length of the dataset analyzed by Cooper and Haltiwanger (2006). Adding more plants does not affect the results.
Table 2: Summary statistics, U.S. plant-level data and models

<table>
<thead>
<tr>
<th>U.S. data</th>
<th>fixed adjustment costs model</th>
<th>fixed information costs model</th>
<th>CH</th>
<th>KT</th>
</tr>
</thead>
<tbody>
<tr>
<td>inaction rate (%): $</td>
<td>i/k</td>
<td>&lt; 1%</td>
<td>8.1</td>
<td>18.3</td>
</tr>
<tr>
<td>negative investment (%): $i/k \leq -1%</td>
<td>10.4</td>
<td>8.8</td>
<td>12.3</td>
<td>n.a.</td>
</tr>
<tr>
<td>positive investment (%): $i/k \geq 1%</td>
<td>81.5</td>
<td>72.8</td>
<td>85.0</td>
<td>n.a.</td>
</tr>
<tr>
<td>positive spike (%): $i/k &gt; 20%</td>
<td>18.6</td>
<td>19.9</td>
<td>18.6</td>
<td>13.2</td>
</tr>
<tr>
<td>negative spike (%): $i/k &lt; -20%</td>
<td>1.8</td>
<td>0.18</td>
<td>1.1</td>
<td>2.3</td>
</tr>
<tr>
<td>serial correlation (corr $[(\hat{i}<em>{t}, \hat{i}</em>{t-1})]$)</td>
<td>0.058</td>
<td>-0.058</td>
<td>-0.061</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Note. For each variable, I compute the time series average for each plant, and report the average across plants.

\[a\] Data are from Cooper and Haltiwanger (2006, table 1).
\[b\] $U = L = 0.15$. \[c\] $\alpha = 0.8$, $r = 0.04$, $\delta = 0.13$, $\sigma = 0.12$
and $\Theta = 0.020$. \[d\] Cooper and Haltiwanger (2006, table 5). \[e\] Khan and Thomas (2008, table II). n.a.: not available.

Table 2 reproduces different moments of plants’ investment rates in the U.S. data (second column) and the models’ predictions (third and fourth column). For the sake of comparison, I also report the moments from two leading papers in the literature, Cooper and Haltiwanger (2006) (fifth column) and Khan and Thomas (2008) (last column).

Here inaction is defined as plant-level investment rates less than 1% in absolute value. Positive investment rates are those at or exceeding 1%, while negative investment rates are those equal to or below −1%. Finally, as is common in this literature, positive spikes are observations in which investment rates exceed 20%, and negative spikes are episodes in which investment rates are less than −20%. The second column of table 2, taken from Cooper and Haltiwanger (2006), documents the nature of capital adjustment behavior using data from the Longitudinal Research Database, a plant-level U.S. manufacturing data set. Clearly, the data exhibit both periods of inactivity and large positive bursts of investment activity, but little evidence of large negative investment episodes. Also, there is a notable asymmetry in positive versus negative investment rates as well as in positive versus negative investment spikes. For instance, positive spikes are observed 10 times as often as negative spikes. Finally, autocorrelation in plant-level investment rates is positive but very low.

The results in the third column show that even the simple model with fixed adjustment costs described in section 3.2 is able to generate investment inactivity at the plant-level as well as both positive and negative
investment spikes.

With only one exception, the inattentiveness model does a reasonably good job. It matches both positive and negative investment of the frequency found in the data. It also produces some inactivity, although it underestimates the frequency of such episodes. Moreover, it generates positive and negative spikes. The exception is that the model produces low negative serial correlation in investment. This moment of the data however warrants a digression, since the findings in the empirical investment literature are mixed.

On the one hand, a key finding in Doms and Dunne (1998) is that large investment episodes are often spread across few (usually two or three) years, that is, there is a high probability of having a spike in the period immediately following a spike. In technical language, adjustment hazard functions – the probability of adjusting – are downward sloping with respect to time since the prior spike. This evidence therefore suggests positive serial correlation of investment at the plant-level and appears supportive of a convex adjustment costs model.

On the other hand, Cooper et al. (1999) argue that unobservable heterogeneity at the plant-level may yield downward-sloping hazard functions even if the hazard for any individual plant is upward sloping. In fact, after controlling for unobservable heterogeneity, they find evidence of upward-sloping hazard functions: the likelihood of a plant experiencing a large investment episode is increasing in time since the previous spike. Put differently, bursts of investment are followed, on average, by periods of low investment, so that the autocorrelation of investment rates may be negative.

Hence the negative serial correlation of the inattentiveness model (and of the non-convex adjustment costs model as well) may be interpreted as evidence that hazard slopes upward. Perhaps not surprisingly, only the hybrid model presented in Cooper and Haltiwanger (2006), which mixes convex and non-convex adjustment costs, is able to reproduce this moment of the data.\footnote{Gelos and Isgut (2001) provide support for the evidence in Doms and Dunne (1998), while Nilsen and Schiantarelli (2003) and Fennema et al. (2006) obtain results consistent with those of Cooper et al. (1999).}

Finally, the last column reports the results in Khan and Thomas (2008). Note that the inattentiveness model does not seem to perform noticeably worse than the Khan and Thomas model, which is “the first to succeed in matching the available moments from the cross-sectional distribution of plant investment rates”, (Khan and Thomas, 2008, p. 408).
So, having established the consistency of the inattentiveness model with the essential features of the microeconomic data, in what follows I aggregate the behavior of many inattentive firms and test whether the model can also fit aggregate investment rate data.

4 Aggregate investment

4.1 Aggregation of the inattentiveness model

This subsection aggregates individual investment decisions to obtain the predictions of the inattentiveness model for the time series of aggregate investment rates. To do so, I treat $Z_t$ as an aggregate profitability shock and I make some assumptions about the distribution of firms’ decision dates.

Each inattentive firm in the economy gathers information and recomputes optimal plans slowly over time. That is, firms respond infrequently – and possibly asynchronously – to the aggregate shock. For each firm, the optimally chosen planning dates $D = \{D(i)\}_{i=0}^{\infty}$ form a sequence of stochastic increasing events. The arrival of decision dates is therefore a stochastic process, whose properties can be described by a set of probability density functions for the length of the inattentiveness period, conditional on when the firm last planned. I denote these by $f_i(t)$ and assume that

**Assumption 1.** The densities $f_i(t)$ describe random variables that are

i) **mutually independent**;

ii) **independent across firms**;

iii) **the same for all firms**;

iv) **uniformly distributed**.

Independence of decision dates allows me to only keep track of the last decision date for each firm. Parts (ii) and (iii) of Assumption 1 in turn allow me to interpret $f_i(t)$ as the actual fraction of firms that are planning at a given point in time. Let $\rho$ denote the mean number of planning dates in a unit of time. As a result, (iv)
implies that, in each period, the share of firms planning is constant and equal to $\rho = \frac{1}{E(d^*)}$. Moreover, at any point in time $t$, the fraction of firms not having planned for $n$ periods, $t - d^* < \forall n < t$, also equals $\rho$.\(^{20}\)

At a given instant in time, aggregate capital equals the sum of the capital amounts chosen by the firms. Letting the index of the firms, $j$, stand for how long it has been since the firm last planned, the aggregate stock of capital is given by

$$K_{t}^{IN,A} = \int_{t-d^*}^{t} \frac{1}{d^*} M^{\alpha} [E_j(Z_t^{1-\alpha})]^{1-\alpha} dj.$$ 

Aggregate investment at time $t$ is given by

$$I_{t}^{IN,A} = \int_{t-d^*}^{t} \frac{1}{d^*} I_{j}^{IN} dj = \frac{1}{d^*} I_t + \int_{t-d^*}^{t} \frac{1}{d^*} I_{j-1}^M dj,$$

where the first term represents the investment gulp of the $\frac{1}{d^*}$ firms that are planning at time $t$, and the second term represents the continuous investment of the remaining inattentive firms. Finally, the aggregate investment rate is defined as the ratio of aggregate investment to aggregate capital stock.

### 4.2 Does the inattentiveness model fit the aggregate data?

I now test the aggregate implications of the model by generating simulated aggregate capital and investment data for a panel of 500 economies over 52 years.\(^{21}\)

Table 3 shows the second and higher order moments of annual aggregate investment rates in the post-war U.S. data (1954-2005, second column) and in the Great Moderation period (1984-2005, third column). It also displays the inattentiveness model’s predictions considering both the entire 52-year sample and the last 22 years of the full sample (fourth and fifth column, respectively). For the sake of comparison, the sixth column reports the moments of the Khan and Thomas (2008) partial equilibrium model.

The serial correlation of aggregate investment rates is about 0.8 in the data, much higher than that observed

\(^{20}\) Caballero and Engel (1991) present a generalized (S,s) model and derive the conditions under which the aggregate distribution is uniform, while Reis (2006b) shows that, under some (very strict) conditions, the arrival of decision dates in the aggregate economy tends to the exponential distribution with parameter $\rho = 1/E(d^*)$. Here I consider the uniform distribution to keep the computational burden manageable.

\(^{21}\) To keep the computational burden manageable, I further set $\Theta = 0.0115$, so that the inattentiveness interval is one year (the remaining parameter values are as in section 3.5).
Table 3: Summary statistics, annual U.S. aggregate data and models

<table>
<thead>
<tr>
<th></th>
<th>U.S. data 1954-2005&lt;sup&gt;a&lt;/sup&gt;</th>
<th>U.S. data 1984-2005&lt;sup&gt;a&lt;/sup&gt;</th>
<th>information costs model&lt;sup&gt;b,c&lt;/sup&gt;</th>
<th>information costs model&lt;sup&gt;b,d&lt;/sup&gt;</th>
<th>KT&lt;sup&gt;e&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>serial correlation</td>
<td>0.797</td>
<td>0.846</td>
<td>0.210</td>
<td>0.172</td>
<td>0.210</td>
</tr>
<tr>
<td>standard deviation</td>
<td>0.010</td>
<td>0.011</td>
<td>0.103</td>
<td>0.102</td>
<td>0.085</td>
</tr>
<tr>
<td>skewness</td>
<td>0.465</td>
<td>0.730</td>
<td>0.223</td>
<td>0.199</td>
<td>1.121</td>
</tr>
<tr>
<td>excess kurtosis</td>
<td>-0.094</td>
<td>-0.446</td>
<td>-0.078</td>
<td>-0.268</td>
<td>2.313</td>
</tr>
</tbody>
</table>

Note. For each moment, I compute the time series average for each economy, and report the average across economies. <sup>a</sup> Data are annual private fixed nonresidential investment-to-capital ratios, computed using Bureau of Economic Analysis tables and following the procedure described in Bachmann et al. (2013, appendix B1). <sup>b</sup> α = 0.8, r = 0.04, δ = 0.13, σ = 0.12 and Θ = 0.0115. <sup>c</sup> Moments are based on a 52-year simulated sample. <sup>d</sup> Moments are computed using the last 22 years of the 52-year sample. <sup>e</sup> Khan and Thomas (2008, table III).

at the plant-level. Aggregate investment rates also exhibit near zero standard deviation, positive skewness and negative excess kurtosis. Note also that, while the second order moments are nearly the same in both periods, the higher moments are larger (in absolute value) during the Great Moderation period.<sup>22</sup>

The performance of the inattentiveness model at the aggregate level is not quite as successful as that at the plant-level. Although aggregation smooths out investment spikes and helps to generate positive serial correlation of investment rates, the model still predicts far too little persistence for the aggregate data. It also overestimates (by roughly 10 times) the volatility of investment rates. Nevertheless, over the other dimensions, the model does a better job. In fact, it predicts about the right amount of skewness and it matches well the excess kurtosis, especially when comparing the model with post-war data.

Despite the mixed performance of the inattentiveness model, one should note that the Khan and Thomas (2008) partial equilibrium model is also not successful in matching these aggregate moments. In fact, their model predicts roughly the same serial correlation and standard deviation as the inattentiveness model, and it also greatly overstates the skewness and kurtosis. It is noteworthy that Khan and Thomas improve the fit of their model when they include the effects of general equilibrium in the lumpy investment environment. In particular, their general equilibrium model yields aggregate investment rates with persistence and

<sup>22</sup> The moments reported in table 3 are different from those reported by Khan and Thomas (2008). Examining the annual private investment-to-capital ratio over the period 1954:2005, Khan and Thomas find persistence, standard deviation, skewness and excess kurtosis of 0.695, 0.008, 0.008 and −0.715, respectively.

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volatility closer to the data. Also motivated by this result, in a companion paper (Verona, 2011) I embed capital investment decisions with inattentiveness in a general equilibrium framework and found an unambiguous improvement in the model fit in moving from the partial equilibrium lumpy investment model to its general equilibrium counterpart. In particular, the persistence of aggregate investment rates increases sharply (although it remains lower than in the data) and its excessive volatility is virtually eliminated in general equilibrium. Intuitively, aggregation helps to smoothen investment spikes, but not sufficiently, and general equilibrium effects (i.e. price movements) further smooth out the excessively large fluctuations in aggregate investment demand and move the model further in the direction of fitting aggregate investment rates moments.

5 Conclusion

In many countries, investment in physical capital at the micro level is lumpy. The most common explanation for this is that firms face non-convex (i.e. fixed) physical adjustment costs, so that they only invest when their existing capital stocks differ excessively from an optimal level, otherwise staying inactive. This paper proposes a novel explanation for this fact: information cost or inattentiveness. Instead of a physical cost, the firm faces a cost of collecting and processing information. This information cost induces the firm to make infrequent investment decisions. On the one hand, in between observation dates, the firm is inattentive and undertakes continuous investment to compensate for depreciation. On the other hand, when the firm does update its information, it reacts to all past and present shocks/news, and the stock of capital jumps to its optimally chosen level. We are thus likely to observe investment spikes at those observation dates. The dynamics of capital adjustment under inattentiveness thus implies lumpy adjustments: periods of low activity interrupted by possibly large capital adjustments.

I found that the inattentiveness model fits the quantitative facts on plant-level investment rates well, and it also matches some (but not all) higher order moments of aggregate investment rates. Moreover, the model does not seem to perform noticeably worse than one of the leading models in the literature. I believe this result is encouraging because this paper is the first to analyze the role of information costs alone in shaping investment in physical capital by firms, in contrast to the mature literature on physical adjustment costs.
The model is somewhat stylized and leaves room for improvement. For example, it abstracts from general equilibrium considerations. In Verona (2011) I embed investment with inattentiveness in the Mankiw and Reis (2006, 2007) sticky information general equilibrium model to examine how inattentiveness alone shapes the business cycle dynamics, as well as to analyze the relevance of lumpy investment for aggregate dynamics. Moreover, in this paper I have treated information costs as a pure alternative to adjustment costs. This allowed me to analyze in detail how information costs alone affect investment dynamics. However, recent works on information frictions and imperfect information in macroeconomics and finance (Alvarez et al., 2011, Bonomo et al., 2011, Abel et al., 2012 and Alvarez et al., 2012,) integrate both adjustment (or transaction) costs and information costs in a single framework. It would be worthwhile to follow this literature and extend this model to include non-convex adjustment costs. Investment in such a model would become time- and state-dependent. Intuitively, the model would predict that the firm today optimally decides when next to collect information and then, according to the state of the economy at that date, decides whether it is optimal to pay the adjustment cost and adjust, or to stay inactive. Both predictions are consistent with the evidence on how firms make decisions in the real world. Moreover, this extension would allow for addressing a richer set of facts on investment that the current version of the model cannot explain (because adjustment is purely time-dependent), such as the procyclicality of investment spikes (see e.g. Gourio and Kashyap, 2007b), the increasing adjustment hazard of investment as a function of mandated investment (see e.g. Caballero et al., 1995) and the conditional heteroscedasticity of aggregate investment rates (see e.g. Bachmann et al., 2013).

Finally, the assumption that the firm does not observe any information when inattentive may be relaxed in two ways. First, one may follow Woodford (2009) and build a hybrid model of rational inattention (frequent but incomplete updating of information) and inattentiveness (infrequent but complete information updating). That is, one could assume that the firm imperfectly observes all information in each period and would incur a discrete cost if it wants to become fully informed about the economy’s state at that moment. Second, it is also possible to follow Mackowiak and Wiederholt (2009) and build a model in which the firm rationally decides to observe and react to some idiosyncratic shocks frequently, and to pay little attention and react slowly to aggregate conditions. This would support the evidence that firms react promptly and aggressively to firm-specific shocks, but with a delay and by small amounts to monetary shocks. I leave these extensions
Appendix A

Quick review of the solution of continuous-time stochastic processes (Brownian motions)

Let \( S_t \) be a geometric Brownian motion defined by the stochastic differential equation

\[
dS_t = \mu S_t dt + \sigma S_t dW_t ,
\]

where \( dW_t \) is a Wiener process, and \( \mu \) and \( \sigma \) are, respectively, the drift and the variance parameter. The behavior of \( S_t \) can be derived by applying Ito’s lemma to \( d \ln S_t \):

\[
d \ln S_t = \frac{1}{S_t} dS_t - \frac{1}{2} \frac{1}{S_t^2} (dS_t)^2 = \frac{1}{S_t} [\mu S_t dt + \sigma S_t dW_t] - \frac{1}{2} \frac{1}{S_t} S_t^2 \sigma^2 dt ,
\]

where the last equality follows from (A.1) and the fact that \((dW_t)^2 = dt\) and \((dt)^2 = dt dW_t = 0\). Thus

\[
d \ln S_t = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dW_t .
\]

Integrating (A.2) and applying the fundamental theorem of calculus yields

\[
S_t = S_0 e^{\left\{ \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W_t \right\}} .
\]

One can also easily derive the following relationships, which hold \( \forall k \):

\[
S_t^k = S_0^k e^{\left\{ k \left( \mu - \frac{\sigma^2}{2} \right) t + k \sigma W_t \right\}} .
\]
\[ E_0(S_t^k) = S_0 e^{\left\{ k \left[ \mu + (k-1) \frac{\sigma^2}{2} \right] t \} } \]  
(A.4)

\[ [E_0(S_t^k)]^{\frac{1}{k}} = S_0 e^{\left\{ k \left[ \mu + (k-1) \frac{\sigma^2}{2} \right] t \} } \]  
(A.5)

**Appendix B**

*The inattentive firm’s problem - section 3.3*

Applying the above properties (A.3 - A.5) to the stochastic processes for \( Z_t (dZ_t = \sigma Z_t dz) \) yields

\[ Z_t = Z_{D(i)} e^{\left\{ -\frac{\sigma^2}{2} [t-D(i)] + \sigma z_t \} } \]

\[ E_{D(i)} (Z_t) = Z_{D(i)} e^{[t-D(i)]} \]  
(B.1)

\[ [E_{D(i)} (Z_{t}^{1-\alpha})]^{\frac{1}{1-\alpha}} = Z_{D(i)} e^{\left\{ -\alpha \frac{\sigma^2}{2} [t-D(i)] \} } \]  
(B.2)

The first order condition with respect to \( K_t \) is

\[ E \left[ \alpha Z_t^{1-\alpha} K_t^{\alpha-1} - (r + \delta) \right] = 0 \Leftrightarrow K_t^{\alpha-1} E (Z_t^{1-\alpha}) = \frac{r + \delta}{\alpha} \Leftrightarrow \]

\[ K_t = \left[ \frac{r + \delta}{\alpha} \right]^{\frac{1}{\alpha}} [E (Z_t^{1-\alpha})]^{\frac{1}{\alpha}} = M^{\frac{1}{\alpha}} [E (Z_t^{1-\alpha})]^{\frac{1}{\alpha}} . \]  
(B.3)

**Proof of proposition 2**

Substituting (B.2) into (B.3) gives the result in the proposition.
Proof of proposition 1

Using (B.3) to evaluate the profit function shows that expected optimal operating profits are

\[ \Pi (x, t) = (1 - \alpha) \left( \frac{r + \delta}{\alpha} \right)^{-\frac{\alpha}{1+\alpha}} \left[ E \left( Z_t^{1-\alpha} \right) \right]^{\frac{1}{1-\alpha}} = \Xi Z_0 \exp \left( -\frac{\sigma^2}{2} t \right), \]

where the second equality follows from (B.2). In this case the Bellman equation (8) may be rewritten as

\[ V (x_0) = \max_d \left\{ \frac{\Xi Z_0 (1 - e^{-(r+b)d})}{r + b} + e^{-rd} \left[ -\Theta \Xi e^{-bd} + A E (Z_d) \right] \right\}, \]

where \( b = \alpha \frac{\sigma^2}{2} \). Given that \( \Pi (x, 0) = \Xi Z_0 \), I make the (educated) guess that the value function is linear: \( V (x) = AZ \), where \( A \) is a coefficient to be determined. The Bellman equation then becomes

\[ AZ_0 = \max_d \left\{ \frac{\Xi Z_0 (1 - e^{-(r+b)d})}{r + b} + e^{-rd} \left[ -\Theta \Xi Z_0 e^{-bd} + A E (Z_d) \right] \right\}. \]

From (B.1), \( E (Z_d) = Z_0 \), and cancelling terms yields

\[ A = \max_d \left\{ \frac{\Xi (1 - e^{-(r+b)d})}{r + b} + e^{-rd} \left[ A - \Theta \Xi e^{-bd} \right] \right\}. \quad (B.4) \]

The first-order condition from the maximization problem is

\[ \frac{\partial A}{\partial d} = e^{-rd} \left\{ \Xi e^{-bd} [1 + \Theta (r + b)] - rA \right\} = 0. \quad (B.5) \]

At the optimum \( d^* \), (B.4) gives the solution for \( A \):

\[
A = \frac{\Xi (1 - e^{-(r+b)d^*})}{r + b} + e^{-rd^*} \left[ A - \Theta \Xi e^{-bd^*} \right]
\]

\[ \Leftrightarrow \]

\[
A = \frac{\Xi (1 - e^{-(r+b)d^*}) - \Theta \Xi (r + b) e^{-(r+b)d^*}}{(r + b) (1 - e^{-rd^*})}.
\]

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Using (B.5) and rearranging then yields the condition

\[
\Gamma (b, \Theta, d^*) = re^{b d^*} - [1 + \Theta (r + b)] \left( r + b - be^{-r d^*} \right)
\]

\[
= re^{\alpha \frac{\sigma^2}{2} d^*} - \left[ 1 + \Theta \left( r + \alpha \frac{\sigma^2}{2} \right) \right] \left( r + \alpha \frac{\sigma^2}{2} - \alpha \frac{\sigma^2}{2} e^{-r d^*} \right).
\]

Next, I check the second-order conditions for the maximization problem in (B.4). Note that

\[
\frac{\partial^2 A}{\partial d^2} = -re^{-rd} \left\{ \Xi e^{-bd} \left[ 1 + \Theta (r + b) \right] - rA \right\}
\]

\[
-b\Xi e^{-(r+b)d} \left[ 1 + \Theta (r + b) \right].
\]

At the optimal \(d^*\), equation (B.5) implies that the first term in the sum is 0, while the second term is always negative. Therefore, \(\frac{\partial^2 A}{\partial d^2} < 0\), which guarantees that the zero of the function \(\Gamma (b, \Theta, d^*)\) corresponds to a maximum.

The optimal choice of inattentiveness \(d^*\) is the zero of \(\Gamma (\cdot)\). For \(\Theta > 0\), \(\Gamma (b, \Theta, 0) = -r \theta (r + b) < 0\),

\[
\Gamma_\Theta = - (r + b) \left[ r + b - be^{-r d^*} \right] < 0 \forall d
\]

\[
\Gamma_d (\cdot) = br \left\{ e^{bd^*} - [1 + \Theta (r + b)] e^{-r d^*} \right\}
\]

\[
\Gamma_d (b, \Theta, 0) = -\Theta br (r + b) < 0; \Gamma_d (b, \Theta, +\infty) = +\infty; \Gamma_d (b, \Theta, d^*) > 0
\]

\[
\Gamma_b (\cdot) = -1 - 2\Theta (r + b) + rde^{bd^*} + e^{-r d^*} [1 + \Theta r + 2\theta b]
\]

\[
\Gamma_b (b, \Theta, 0) = -\Theta r < 0; \Gamma_b (b, \Theta, +\infty) = +\infty; \Gamma_b (b, \Theta, d^*) > 0.
\]

For \(d^* > 0\), the implicit function theorem implies that

\[
\Gamma_\Theta (b, \Theta, d^*) + \Gamma_d (b, \Theta, d^*) \frac{\partial d^*}{\partial \Theta} = 0 \Leftrightarrow \frac{\partial d^*}{\partial \Theta} = -\frac{\Gamma_\Theta (b, \Theta, d^*)}{\Gamma_d (b, \Theta, d^*)} > 0
\]

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\[ \Gamma_b(b, \Theta, d^*) + \Gamma_d(b, \Theta, d^*) \frac{\partial d^*}{\partial b} = 0 \iff \frac{\partial d^*}{\partial b} = -\frac{\Gamma_b(b, \Theta, d^*)}{\Gamma_d(b, \Theta, d^*)} < 0. \]

Finally, to obtain the approximation, let me define \( \tilde{\Theta} = \sqrt{\Theta} \) so that

\[ \Gamma_b(b, \tilde{\Theta}, d^*) = r e^{bd^*} - \left[ 1 + \tilde{\Theta}^2 (r + b) \right] \left( r + b - be^{-rd^*} \right) \]

and note that \( \Gamma(b, 0, 0) = 0, \Gamma_d(b, 0, 0) = 0 \) and \( \Gamma_d(b, 0, 0) = 0. \) The implicit function theorem, \( \Gamma_{\tilde{\Theta}} + \Gamma \frac{\partial d^*}{\partial \tilde{\Theta}} = 0 \) therefore does not apply since \( \Gamma_{\tilde{\Theta}} = \Gamma_d = 0, \) so the point \( \tilde{\Theta} = d = 0 \) is a bifurcation point. One further round of differentiation plus the fact that \( \Gamma_{d\tilde{\Theta}} (b, 0, 0) = 0 \) lead to the conclusion that

\[ \Gamma_{\tilde{\Theta}} + \Gamma_{dd} \left( \frac{\partial d}{\partial \tilde{\Theta}} \right)^2 = 0 \implies \frac{\partial d}{\partial \tilde{\Theta}} = \sqrt{-\frac{\Gamma_{\tilde{\Theta}}}{\Gamma_{dd}}} . \]

Since \( \Gamma_{d\tilde{\Theta}} (\cdot) = -2 (r + b) (r + b - be^{-rd^*}) \) and \( \Gamma_{dd} (\cdot) = br \left\{ be^{bd} + r \left[ 1 + \tilde{\Theta}^2 (r + b) \right] e^{-r d^*} \right\}, \) \( \Gamma_{d\tilde{\Theta}} (b, 0, 0) = -2r (r + b) \) and \( \Gamma_{dd} (b, 0, 0) = br (r + b), \) so that

\[ \frac{\partial d}{\partial \tilde{\Theta}} = \sqrt{-\frac{\Gamma_{\tilde{\Theta}}}{\Gamma_{dd}}} = \sqrt{\frac{2}{b}} . \]

Since a first-order Taylor approximation of \( d^* \) around \( \tilde{\Theta} = 0 \) is given by \( d^* = \frac{\partial d^*}{\partial \tilde{\Theta}} \sqrt{\Theta}, \) then

\[ d^* = \frac{\sqrt{4\Theta}}{\alpha \sigma^2} . \]
References


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