Investment, Tobin's Q, and cash flow across time and frequencies*

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Abstract

The investment literature has long acknowledged the time- and frequency-varying dynamics of the relationship between investment, Tobin's Q and cash flow. In this paper, we use continuous wavelet tools to estimate and assess the relationship between these variables simultaneously at different frequencies and over time. We find that i) Q and cash flow are complementary sources of information for investment, the former being more important for firms' investment decisions in the medium-to-long run and the latter at business cycles frequencies; and ii) investment-Q sensitivity declines over time at all frequencies, while investment-cash flow sensitivity declines at business cycles frequencies but remains largely stable over the medium-to-long run.

Keywords: investment, Tobin's Q, cash flow, time-frequency estimation, continuous wavelets

JEL codes: C49, E22, G31

1 Introduction

According to the original Q theory of investment proposed by Tobin (1969), corporate investment is an increasing function of average Q, the market valuation of a firm divided by the replacement cost of its capital stock. However, if Q fails to control for the entire investment opportunity set (*i.e.* if Q is an imperfect measure of both short- and long-run investment expectations and decisions), then other variables could act as useful instruments in investment regressions. Over the years, several papers, both empirical (*e.g.* Bond and Van Reenen, 2007) and theoretical (*e.g.* Abel and Eberly, 2011), have indeed demonstrated that a number of variables (mainly representing liquidity and finance constraints) are useful in explaining aggregate investment. Furthermore, investment spending of firms is likely to be sensitive not only to the availability of external equity finance (as proxied by Q), but also of internal funds (cash flow) and external debt finance (bank loans and corporate bonds). Hence, other financial variables beyond Q likely influence investment decisions.

Empirical evidence has shown that the investment sensitivities to Q and cash flow vary dynamically and non-linearly over time. For instance, McLean and Zhao (2014) find that investment-Q (investment-cash flow) sensitivity varies over the business cycle, with higher sensitivity during expansions (recessions). Several authors (see *e.g.* Lettau and Ludvigson, 2002 and Abel and Eberly, 2012) have also suggested that Q and liquidity variables may not be related to investment in the same way at all frequencies and horizons. For instance, Abel and Eberly (2012) argue that movements in Q could primarily affect investment over long-horizons into the future, as Q (based on equity prices) is a forward-looking variable that captures information about the value of long-term growth options available to the firm. Hence, Q may not be very informative about near-term investment plans and thus could perform poorly in explaining current investment. Likewise, liquidity variables mainly reflect current technology, productivity and demand, so current cash flow, sales or profits could predict short-run investment better than long-term investment.

In the light of this empirical evidence, three features should be taken into account when estimating the equation linking investment, Q and cash flow. First, investment depends on several variables. Even if a variable is potentially a sideshow for aggregate investment (as Grullon, Hund, and Weston, 2018 argue for Q), it is nevertheless important to control for it as it could affect the investment sensitivity with regard to the other variable. Second, the relationships between these variables are time-varying. Third, the relationships are frequency-dependent. Against this background, the main contribution of this paper is to analyze the time-and-frequency-varying role that Q and cash flow have played as drivers of aggregate investment dynamics. We do so by using the continuous wavelet transform tools developed by Aguiar-Conraria, Martins, and Soares (2018). These tools allow us to estimate investment-Q and investment-cash flow sensitivities over time and across frequencies simultaneously in a multivariate setting.

Our first main result is that the information content of Q and cash flow are complementary rather than alternative to each other, since Q and cash flow relate to investment at different frequencies. Namely, we find a positive, stable medium-to-long-run relationship between investment and Q, while the positive and stable relationship between investment and cash flow is largely confined at business cycles frequencies. The second main result concerns the time-variation of the investment sensitivities to Q and cash flow. We find that investment-Q sensitivity declines over time at all frequencies, while investment-cash flow sensitivity declines at business cycles frequencies but remains largely stable at lower frequencies.

These findings are similar to those obtained using the financial accelerator model of Bernanke, Gertler, and Gilchrist (1999), whereas the availability of credit depends on the firm's leverage, which in turn fluctuates endogenously over the business cycle. Over the long run, however, investment is driven by the fundamental value of the firm's projects, thus firm's investment becomes less sensitive to the availability of finance. These results also highlight the importance of looking beyond business cycle fluctuations and support the view that policymakers should pay attention to medium/long run cycles when conducting stabilization policies (Comin and Gertler 2006; Lubik, Matthes, and Verona 2019).

The rest of the paper is organized as follows. Section 2 reviews the two strands of literature on which this paper builds. The first strand deals with time-and-frequency-varying dynamic behaviors of investment-Q and investment-cash flow sensitivities. The second uses wavelet tools in the analysis of time-and-frequency-varying relationships between macro and financial variables. The basic definitions and the intuition of the underlying concepts of the continuous wavelet transform are reported in section 3. The data are described in section 4. The investment equation is estimated in the time-frequency domain in section 5. Section 6 concludes.

2 Related literature and contribution

This paper is related to the literature on the time-and-frequency-varying features of the relationship between investment, Q and cash flow, as well as the literature on the theory and economic applications of wavelet tools to estimation of a parametric function in the time-frequency domain. In this section, we provide a brief overview of these two strands of the literature.

2.1 The investment equation: $I = f\ (Q\,,\, cash\, flow)$

2.1.1 Time-varying investment sensitivities

Several empirical papers document how investment-Q and investment-cash flow sensitivities vary over time.[1]

Looking at low-frequency movements (long-run relationships), Brown and Petersen (2009) argue that the sensitivity of investment to Q has declined since the 1970s, while Agca and Mozumdar (2008) and Chen and Chen (2012) suggest that it has fluctuated over time but overall remained rather stable. The empirical evidence is also mixed as regards investment-cash flow sensitivity. Using micro-level data, several studies (*e.g.* Agca and Mozumdar, 2008; Brown and Petersen, 2009; Chen and Chen, 2012; Lewellen and Lewellen, 2016) find that the cash flow sensitivity of investment has steadily declined since the 1970s. Further, Chen and Chen (2012) show that such sensitivity vanished completely during the 2007-2009 credit crunch. In contrast, using data for the

[1] Several studies (*e.g.* Covas and Haan, 2011, Jermann and Quadrini, 2012 and Begenau and Salomao, 2019) analyze how internal and external firm financing varies over the business cycle.

largest 100 investing firms in the US, Grullon, Hund, and Weston (2018) report that the effect of cash flow on aggregate investment actually increased over the last 30 years (and, where significant, Q had an unexpected negative effect on investment).

As regards business cycle fluctuations, McLean and Zhao (2014) find that investment is more sensitive to Q (cash flow) during business-cycle expansions (recessions), as well as during periods of high (low) investor sentiment.

Finally, Gallegati and Ramsey (2013b) find evidence of structural instability of the investment-Q relationship, in that the investment-Q sensitivity is negative during the 1980s and positive (as expected) during other observed periods.

Overall, the pattern of variation and instability of the Q- and cash flow-sensitivity of investment detected in the literature strongly motivates the use of an empirical method such as continuous wavelets as it allows study of the relationships between variables without losing the information about the time-varying dynamics underlying these relationships at all relevant cyclical frequencies.

2.1.2 Frequency-dependent relationships

There is no reason to believe that economic variables relate similarly to each other at all frequencies. Indeed, macroeconomic theory suggests that some of the most important aggregate relationships (*e.g.* the permanent income hypothesis, the Phillips curve and the quantity theory of money) are frequency-dependent. Specifically, these relationships are different in sign, magnitude or both for low-frequency fluctuations (long run) and high-frequency fluctuations (short run).

The idea of a long-run relationship between investment and Q dates back to Keynes

(1936), who noted that investment planning and decisions are based on expectations of the prospective yield of an investment over a span of years. Engle and Foley (1975) later suggested that most of the power in the relationship between investment and Q is found at low frequencies, and thus a long-run relationship.

More recently, several authors have argued that Q and cash flow may not be related to investment in the same way at all the frequencies and horizons as the information they provide could be, at least to some extent, complementary. We highlight the following contributions to the literature.

On the theoretical side, Abel and Eberly (2012) develop a model in which investment is more responsive to Q at long horizons than at short horizons. Likewise, cash flow only affects current investment and not future investment. Cao, Lorenzoni, and Walentin (2019) build a model in which investment is sensitive to current profitability, *i.e.* the determinant of current internal financing, while Q is relatively more sensitive to profitability and growth further in the future. Overall, theory and models suggest that Q and cash flows are both indicators of future firm profitability, with cash flow having predictive power for short-term profitability and Q for the long run.

On the empirical side, using the present-value approach to Q, Price and Schleicher (2005) find that Q is positively related to investment and capital growth over medium and long horizons. More recently, two proxies for Q have been proposed in the literature as better predictors of investment than the standard measure of Q based on equity prices (in terms of the size and significance of the regression coefficient and fit of the investment regression). Both measures of Q do a good job of capturing the effects on investment at different horizons and frequencies. First, Gallegati and Ramsey (2013a) show that the measure of Q proposed by Philippon (2009), which is based on corporate bond prices, is a better proxy than equity market Q because its four individual components have different explanatory power at different time horizons for aggregate investment (two relate to investment in the short run, and two are associated to long-term investment movements). Second, Kilponen and Verona (2016) show that the measure of Q proposed by Peters and Taylor (2017), which includes intangible capital, is a better predictor of total investment (the sum of tangible and intangible investment) because it effectively captures the effects on total investment at business-cycle frequencies and over the long run.[2]

Wavelet analysis is a straightforward way of disentangling and analyzing how variables interact at different frequencies (and how this interaction has evolved over time).

2.2 Wavelets in economics

Wavelets have long been popular in fields such as geophysics, engineering, medicine and biomedical engineering. Notably, the French mathematician Yves Meyer received the 2017 Abel Prize "for his pivotal role in the development of the mathematical theory of wavelets".[3]

The continuous wavelet transform (CWT) is becoming a popular tool in econometric analysis. The most common argument for using the CWT is the possibility of *simultaneously* assessing how variables are related at different frequencies and how these

[2] Besides these two alternative measures of Q, other proxies for Q have been computed using tax and regulatory policy variables (McGrattan and Prescott, 2005), labor market variables (Merz and Yashiv, 2007 and Mumtaz and Zanetti, 2015, 2016) and option prices (Celil and Chi, 2016).

[3] The Abel Prize is, along with the Fields Medal, considered to be the highest honors a mathematician can receive. These awards have often been described as the Nobel Prize for mathematics.

relationships have changed over time. For instance, the CWT is well suited to study the interaction between variables (macro/financial) and cycles (business/credit) that fluctuate at different frequencies (see Verona, 2016).

The papers closely related to this work are Gallegati and Ramsey (2013a,b, 2014), Kilponen and Verona (2016) and Aguiar-Conraria, Martins, and Soares (2018). Using CWT tools, Gallegati and Ramsey (2013a) analyze the time-frequency correlation between investment and two measures of Q (equity Q and bond Q). They do not, however, estimate the intensity of the investment-Q relationship or how it changes when controlling for cash flow, and they are silent about the investment-cash flow sensitivity. Gallegati and Ramsey (2013a,b, 2014) apply the maximal overlap discrete wavelet transform (MODWT) to decompose the variables into their different frequency components and estimate by ordinary least squares (OLS) a sequence of investment regressions. Following their lead, Kilponen and Verona (2016) apply the MODWT decomposition to several measures of Q and investment, and then run an out-of-sample time-frequency forecasting exercise. The shortcoming of the MODWT decomposition is that the investment-Q and the investment-cash flow sensitivities in each frequency band are constant over the sample period, so it is not possible to analyze their time-varying features.[4]

The CWT tools recently developed by Aguiar-Conraria, Martins, and Soares (2018) allow for overcoming these problems, making it possible to estimate multivariate equations (in their case, a Taylor rule relating the nominal interest rate with the inflation

[4] Other contributions using CWT analysis include Rua (2012), Aguiar-Conraria, Martins, and Soares (2012, 2013), Verona (2016) and Aguiar-Conraria, Soares, and Sousa (2018) (the latter paper also makes use of the concepts of partial phase-differences and partial gains). The early papers in economics using wavelets (Ramsey and Lampart, 1998a,b) rely on the discrete wavelet transform. Recent papers using the discrete wavelet transform include Gallegati, Gallegati, Ramsey, and Semmler (2011), Crowley and Hallett (2018) and Faria and Verona (2018).

rate and the output gap) in the time-frequency domain. These tools allow to assess the intensity, significance, sign and synchronization of the co-movements of investment, Q and cash flow at different frequencies. Hence, we can estimate and analyze the investment-Q and the investment-cash flow sensitivities across time and frequencies.

The advantages of wavelets over standard econometric techniques

Traditional econometric techniques (time series and frequency analysis) impose strong assumptions about the data generating process as they require a variable to be stationary. However, several economic and financial time series are clearly non-stationary as they exhibit *e.g.* trends, structural breaks and volatility clustering.

Unlike Fourier analysis, wavelets are defined over a finite window in the time domain, which is automatically resized according to the frequency of interest. That is, using a short time window allows to isolate the high-frequency movements of a time series, while looking at the same signal with a large time window reveals its low-frequency dynamics. Hence, by varying the size of the time window, one can analyze simultaneously both time-varying and frequency-varying features of the time series. Wavelets are thus useful when dealing with non-stationary time series.

One may ask the question of why not just use the more popular Baxter and King (1999) band-pass filter. This filter is a combination of a Fourier decomposition in the frequency domain with a moving average in the time domain. It is optimized by minimizing the distance between the Fourier transform and an ideal filter. Like the short-time Fourier transform, it is an "optimal" Fourier filtering on a moving window in the time domain with constant length regardless of the frequency being extracted. Wavelet filtering, in contrast, provides better resolution in the time domain as the wavelet basis functions are both time-localized and frequency-localized.

3 The continuous wavelet transform

In this section, we introduce the continuous wavelet transform. The aim is to provide the reader with some basic definitions and an intuition of the underlying concepts. We closely follow the notation and description of Aguiar-Conraria and Soares (2014) and Aguiar-Conraria, Martins, and Soares (2018). We refer the reader to those papers for further technical details and for a review of economic and financial applications of the CWT.

3.1 Univariate and bivariate wavelet tools

A wavelet $\psi(t)$ is a function of finite length that oscillates around the t-axis. Like a propagating wave, it loses power as it moves away from the center. The name wavelet originates from the admissibility condition that requires the mother wavelet to be of finite support (small) and oscillatory (wavy) behavior, hence wavelet (small wave). The most commonly used mother wavelet in economic applications (and the one used here) is the Morlet wavelet $\psi(t) = \pi^{-\frac{1}{4}} e^{6it} e^{-\frac{t^2}{2}}$. This specific choice yields a simple relation between scale s and frequency f, with $f \approx 1/s$.

Let *x(t)* be a time series. The continuous wavelet transform of *x(t)* with respect to a given mother wavelet ψ can be written as

$$W_x(\tau, s) = \frac{1}{\sqrt{\mid s \mid}} \int_{-\infty}^{+\infty} x(t)\, \overline{\psi}\left(\frac{t-\tau}{s}\right) dt \ ,$$

where the bar denotes the complex conjugate, the location parameter τ determines the position of the wavelet along the t-axis, and the scale parameter s defines how the mother wavelet is stretched. Scale is inversely related to frequency: a lower (higher) scale means a more (less) compressed wavelet, allowing us to focus on the higher (lower) frequencies of the time series.[5]

As all the variables we introduce are functions of τ and s, we omit τ and s in the formulas for convenience.

The wavelet power spectrum (WPS) of *x(t)* is defined as $WPS_x = \mid W_x \mid^2$, and measures the local variance distribution of the time series *x(t)* around each time and scale/frequency. The global wavelet power spectrum (GWPS) is obtained by averaging the WPS over all times: $GWPS_x = \int_{-\infty}^{\infty} \mid W_x \mid^2 d\tau$.

Given two time series *y(t)* and *x(t)*, the cross-wavelet transform of y and x is defined as $W_{yx} = W_y \overline{W_x}$, where W_y and W_x are the wavelet transforms of y and x, respectively. The absolute value of W_{yx} is usually referred to as the cross-wavelet power and depicts the local covariance between the two series in the time-frequency space. The wavelet coherency of y and x is given by

$$R_{yx} = \frac{\mid S(W_{yx}) \mid}{\left[S(\mid W_y \mid^2)\, S(\mid W_x \mid^2)\right]^{1/2}} \ , \tag{1}$$

[5] The wavelet transform can also be expressed in polar form as $W_x(\tau, s) = \mid W_x(\tau, s) \mid e^{i\phi_x(\tau,s)}$, where the angle $\phi_x(\tau, s)$, $-\pi < \phi_x(\tau, s) \leq \pi$, is referred to as the wavelet phase.

where S denotes a smoothing operator in both time and scale (without smoothing, wavelet coherency would be always equal to one as in the Fourier analysis). The wavelet coherency measures the strength of the relationship between y and x around each point in time and for each frequency. It can thus be considered a direct measure of the local correlation between two time series in the time-frequency space. R_{yx} varies between 0 and 1, with a high (low) value indicating a strong (weak) co-movement between the variables.

The wavelet gain of *y(t)* over *x(t)* is defined as

$$G_{yx} = \frac{\mid S\left(W_{yx}\right) \mid}{S\left(\mid W_x \mid^2\right)} \tag{2}$$

and can be interpreted as the absolute value of the regression coefficient in the regression of y on x at each time and frequency.

Finally, the wavelet phase-difference provides information about the lead-lag relationship between the two series, as well as about the sign of the wavelet gain given by equation (2), as a function of time and frequency. It is given by

$$\phi_{yx} = \arctan \frac{\Im\left[S\left(W_{yx}\right)\right]}{\Re\left[S\left(W_{yx}\right)\right]} \ ,$$

where $\Re$ and $\Im$ are the real and imaginary parts of W_{yx}, respectively, and $-\pi \leq \phi_{yx} \leq \pi$. Variables are in phase, *i.e.* they move together, if $-\pi/2 < \phi_{yx} < \pi/2$, with y leading if $0 < \phi_{yx} < \pi/2$ and x leading if $-\pi/2 < \phi_{yx} < 0$. Otherwise, they are out of phase, *i.e.* negatively correlated, with y leading if $-\pi < \phi_{yx} < -\pi/2$ and x leading if $\pi/2 < \phi_{yx} < \pi$.

As the CWT at any point in time uses information of neighboring data points, the values of the wavelet transform at the beginning and end of the sample are always incorrectly computed as the time series have to be padded *e.g.* with zeros. The region in which the CWT suffers from such edge effects, which is larger the lower the frequencies, is known as the cone of influence. Results should be interpreted carefully in this region of the time-frequency space.

3.2 Multivariate wavelet tools (the case of three variables)

When investigating the relationship between two variables, it might be needed to control for their interactions with other variables when calculating coherency, gains and phase-differences. This is precisely our case as we aim at assessing the sensitivity of investment to Q and cash flow (along time and frequencies). We present here the formulas for the multivariate wavelet tools – multiple wavelet coherency, partial wavelet coherency, partial wavelet phase-difference and partial wavelet gain – for the case in which we have three series (y, x and z). The formulas for the general case can be found in Aguiar-Conraria, Martins, and Soares (2018, appendix A).

Let $\varrho_{yx} = \frac{S(W_{yx})}{[S(|W_y|^2)S(|W_x|^2)]^{1/2}}$ denote the complex wavelet coherency of y and x and σ_y denote the square root of the smoothed WPS of series y: $\sigma_y = \sqrt{S\left(\mid W_y \mid^2\right)}$.

The squared multiple wavelet coherency between the series y and the two series x and z, denoted by $R^2_{y(xz)}$, is given by

$$R^2_{y(xz)} = \frac{R^2_{yx} + R^2_{yz} - 2\Re\left(\varrho_{yx}\varrho_{xz}\overline{\varrho_{yz}}\right)}{1 - R^2_{xz}} \quad ,$$

where R^2_{yx} is the squared of the wavelet coherency (given by equation (1)). The multiple wavelet coherency between y and the two series x and z is the positive square root of R^2_{yx} and represents the time-frequency analog of the R^2 in the typical multivariate regression of y on x and z.

The complex partial wavelet coherency between y and x, after controlling for the third series z, is denoted by $\varrho_{yx.z}$ and given by

$$\varrho_{yx.z} = \frac{\varrho_{yx} - \varrho_{yz}\overline{\varrho_{xz}}}{\sqrt{\left(1 - R^2_{yz}\right)\left(1 - R^2_{xz}\right)}} \quad .$$

The absolute value and the angle of $\varrho_{yx.z}$, are respectively referred to as the partial wavelet coherency and the partial wavelet phase-difference between y and x, after controlling for z. The partial wavelet coherency captures the co-movement between y and x, filtering out the effect of z.

Finally, the partial wavelet gain between y and x, after controlling for z, is denoted by $G_{yx.z}$ and given by

$$G_{yx.z} = \frac{\mid \varrho_{yx} - \varrho_{yz}\overline{\varrho_{xz}} \mid}{1 - R^2_{xz}} \frac{\sigma_y}{\sigma_x} \quad .$$

It can be interpreted as the absolute value of the regression coefficient in the regression of y on x, after controlling for z.[6]

[6] It is still an open question how to appropriately obtain confidence intervals for the (partial) gain. Hence, the analysis of the gains should be complemented by inspecting coherencies, and one should only focus on the regions whose (multiple and partial) coherencies are statistically significant.

4 Data

We use quarterly aggregate US data from 1952:Q1 to 2017:Q4. The investment rate is the ratio of aggregate (private nonresidential fixed) investment to aggregate capital for the whole economy. Tobin's Q is calculated as the ratio of market value of non-financial corporate equities to net worth, where net worth is given by total assets minus liabilities. For cash flow, we follow Philippon (2009) and use the ratio of corporate profits over GDP.[7]

The graphs in the left column of figure 1 show the time series of the variables, together with NBER recessions (gray-shaded areas) and with the sample average value of the variable (black horizontal line). The graphs in the middle and right columns show the global wavelet power spectrums (GWPS) and the wavelet power spectrums (WPS) of the variables, respectively. The WPS measures the variance of the series at each time-frequency locus. In the WPS charts (as well as in the coherency charts in the following section), time is on the horizontal axis and period cycles (in years) on the vertical axis, hotter colors (yellow and red) correspond to higher volatility/coherency and colder colors (green and blue) to lower volatility/coherency, white lines denote the local maxima of the WPS (an estimate of the cycle periods), black (gray) contours mark significance at the 5% (10%) level, and black dashed lines denote the cone of influence (*i.e.* the region affected by edge effects).[8] The GWPS gives the average wavelet power

[7] The series for investment rate is from Amit Goyal's website (http://www.hec.unil.ch/agoyal/docs/PredictorData2017.xlsx). The data for Tobin's Q is from Flow of Funds Table B103 – Balance Sheet of Non-financial Corporate Business. Corporate profit data are taken from NIPA Table 1.12 and GDP data from FRED.

[8] Following Aguiar-Conraria, Soares, and Sousa (2018), significance levels for the WPS were computed using a chi-squared distribution as the theoretical distribution, assuming a flat spectrum as the null. In the coherency charts, significance levels were obtained by bootstrapping with 5000 replications.

for each frequency. It is obtained by averaging the WPS over all times. This makes it directly comparable with classical spectral methods.

The investment rate is pro-cyclical, exhibiting slow booms followed by quick busts. The most notable investment boom is the one starting in the mid-1990s, which characterizes the longest US expansionary period in the post-World War II era. This investment boom ends just before the collapse of the dot-com bubble in 2001. Investment also falls remarkably during the Great Recession and stays below its sample mean thereafter. The WPS of the investment rate is high throughout the entire sample at frequencies around 16 years (consistent with the peak in the GWPS at that frequency), as well as at lower frequency (roughly 30-year cycle) up to 2000. There is also evidence of another cycle, starting from 1965, at a frequency of about 8 years, which gradually shortens and loses importance before disappearing in 1985. At about the same time, a longer cycle (about 10 years) starts and slowly becomes an 8-year cycle toward the end of the sample.

In addition to high-frequency fluctuations, Q exhibits large and low-frequency swings, moving upwards until 1968, downwards to 1982 and upwards again until the bursting of the dot-com bubble in early 2000. After that, Q seems stabilize, despite the large fluctuations associated with the Great Recession and the following recovery. The WPS indicates that the volatility of Q occurs mainly at low frequencies (roughly 30-year cycle) throughout the entire sample.[9] From 1985 to 2005, volatility also increases at higher frequencies, with two cycles with periods of about 6 and 16 years appearing in

Furthermore, we use constant padding boundary conditions.

[9] Groth and Madsen (2016) point out that medium-term movements (cycles of 8-40 years duration) in investment and Q overshadow the movements at business-cycle frequencies.

those decades.

Cash flow displays a slow downward trend until 1983. A positive trend emerges in early 1990s, despite two large drops associated with the early 2000s recession and the Great Recession. Cash flow leads the business cycle as it usually peaks a few quarters before the beginning of the subsequent business cycle recession. The volatility of cash flow occurs mainly at frequencies between 8 and 16 years throughout the entire sample, with the main cycle featuring a period of about 12 years at the beginning of the sample that gradually becomes a 8-year cycle toward the end of the sample.

The evidence from the WPS that the bulk of the volatility of Q is concentrated at much lower frequency than the volatility of cash flow (30 versus 10 years) already hints that these two variables could be related to investment at different frequencies.

Having provided the time-frequency description of the variables, we now move to the results of the empirical estimation of the investment equation using multivariate CWT tools.

5 The investment equation in the time-frequency domain

To assess the strength of the relationship between investment and our other two variables (Q and cash flow), we first look at the multiple coherency, *i.e.* the time-frequency analog of the R^2 in a typical regression. We then disentangle the relative impact of each financial variable on investment by analyzing the partial coherency, the partial phase-difference and the partial gain between investment and each of the financial variable in the investment equation, controlling for the effects of the other.[10] In particular, the partial gains give the estimate of the modulus of the coefficients associated with each financial variable in the investment equation, allowing for their variations along time and across frequencies. The partial phase-differences give the estimate of the signs of the coefficients associated with each financial variable (and the information about the lead-lag between the variables) in the investment equation at each point in time and for different frequencies.

For the sake of exposition, we present partial phase-differences and partial gains for three frequency intervals, namely for cycles of period 1.5-4 years (shorter business cycle frequencies), cycles of period 4-8 years (longer business cycle frequencies) and cycles of period 8-32 years (the medium-to-long run).[11]

5.1 Overall fit of the investment equation

The top left graph in figure 2 displays the multiple coherency between investment, Q and cash flow, which provides the overall fit of our multivariate investment equation in the time-frequency domain. Specifically, Q and cash flow are jointly significant explanatory variables of investment in those time-frequency regions with a significant multiple coherency (red and statistically significant areas).

The fit of the investment equation is overall rather good and it shifts gradually toward cycles of longer length. Up to 1965, the multiple coherency is significant only at shorter

[10] Adding (or replacing cash flow with) other financial variables such as bank loans or corporate bonds does not qualitatively affect the results.

[11] For the partial phase-differences and partial gains we obtain similar results by splitting the medium-to-long run into cycles of periods 8-16 and 16-32 years.

business cycle frequencies, while after 1965 it becomes significant also at cycles up to 16-year period. At medium-to-long-run cycles (about 30-year period), multiple coherency is always statistically significant, which suggests a medium-to-long-run relationship between investment and the two financial variables. Only three regions are characterized by low coherency: up to 1970 at longer business cycle frequencies, between the 1990s and the 2000s at business cycle frequencies, and up to 1985 at frequencies between 16 and 30 years.

5.2 Investment and Q

The graphs in the middle row of figure 2 report the partial coherency between investment and Q (after filtering out the effect of cash flow), as well as the partial phase-differences and the partial gains of Q in a time-frequency regression of investment on Q and cash flow.

The partial coherency (left column) shows that, first and foremost, the correlation between investment and Q is a medium-to-long-run phenomenon (cycles of more than 24 years). That is, there is a stable, medium-to-long-run relationship between investment and Q throughout the entire sample period. Occasionally, there are other regions with a strong correlation between them: at shorter business cycle frequencies from 1960 to 1990 and after 2000, at longer business cycle frequencies from 1990 to 2005, and at cycles between 8 and 16 years from 1980 to 2000.

The partial gains, reported in the right column, portray the modulus of the regression coefficient at each time and frequency. They inform about the magnitude of the impact that a shock in Q will have on investment after controlling for cash flow. Recall that they are affected by edge effects, so that their values at the beginning and at the end must be carefully interpreted. The short-run business cycle gain gradually increases from the beginning of the sample until around 1985, then decreases sharply and stabilizes at a lower level (without though reaching its peak again). The longer business cycle and medium-to-long-run coefficients peak at the beginning of the sample (at around 1968), and then slowly decrease. Overall, the investment-Q sensitivity declines over time throughout the sample period in all frequency bands. Furthermore, with the exception of the 1980-1986 period, the longer business cycle and medium-to-long-run investment-Q sensitivities are larger than the shorter business cycle sensitivity. This suggests that investment is more responsive to Q at lower frequencies and thus reinforces the findings of the partial coherency analysis that the power in the relationship between investment and Q is found in the medium-to-long run.

According to the phase-differences (graphs in the middle column), at longer business cycle frequencies and in the medium-to-long run investment and Q are usually in phase (the phase-difference is usually between $-\pi/2$ and $\pi/2$), which implies a positive relationship between investment and Q in these frequency bands. Furthermore, Q leads investment (phase-difference between $-\pi/2$ and 0). At short-run frequencies (period of 1.5-4 years), the relationship is usually positive, with Q leading investment. The sole exception is the decade around the 1990s, when there is a negative relationship between them.[12] This finding comports with those of Gallegati and Ramsey (2013b), who find a negative (contemporaneous) sensitivity of investment to Q around that period.

Overall, these results suggest that Q is an important determinant of aggregate invest-

[12] As one would expect, the medium-to-long-run gain and phase-difference display smoother dynamics as they capture the low-frequency relationship between variables.

ment even after controlling for cash flow. This contradicts the findings of Grullon, Hund, and Weston (2018), who argue that Q is a sideshow at the aggregate level. Furthermore, and in line with the findings of Gallegati and Ramsey (2013a, 2014), the leading behavior of Q at all frequencies confirms that Q is a forward-looking variable that does a good job in capturing the expected present discounted value of a firm's longer-term profits, and hence the firm's longer-term investment decisions.

5.3 Investment and cash flow

The graphs in the bottom row of figure 2 report the partial coherency between investment and cash flow (after controlling for Q), as well as the partial phase-differences and the partial gains of cash flow in a time-frequency regression of investment on Q and cash flow.

According to the partial coherency (left column), the correlation between investment and cash flow is high and statistically significant mainly at frequencies between 4 and 16 years. Remarkably, there is low coherency at fluctuations longer than 16 years throughout the entire sample period. There are also some periods when the correlation is high and statistically significant also at short-run business cycle frequencies (1.5-4 years), *e.g.* in the decades around the 1960s, 1980s and 2000s.

The partial phase-differences (graphs in the middle column) reveal that relationship between investment and cash flow is positive regardless of the frequency, with cash flow leading investment at longer business cycle frequencies and in the medium-to-long run (which are the frequency bands characterized by high and statistically significant coherency).

As regards the dynamics of the investment-cash flow sensitivities (right column), the short-run business cycle sensitivity displays an oscillatory behavior around a slow declining trend over time. In particular, after the peak in the early 2000s, the sensitivity drops significantly and never recovers. At longer business cycles frequencies, the sensitivity is rather stable from 1970 until the early 1990s, then it suddenly drops and only partially recovers until the end of the sample. In the medium-to-long run, the investment-cash flow sensitivity has been overall stable throughout the entire sample period. These results contrast the findings in Grullon, Hund, and Weston (2018), who find that the investment-cash flow sensitivity of the largest 100 investing firms has increased over time. Finally, the business cycle investment-cash flow sensitivities are usually larger than the medium-to-long-run one, thus suggesting that investment is more responsive to cash flow at business cycles frequencies. This, in combination with the partial coherency results, indicates that the power in the relationship between investment and cash flow is concentrated at typical business cycles frequencies.

6 Conclusions

Economists have long understood that some relationships between economic variables are time varying, frequency varying or both. In particular, the empirical and theoretical investment literature has long acknowledged the time- and frequency-varying dynamics of the relationship between corporate investment, Tobin's Q and cash flow. In this paper, we estimated the investment equation using the continuous wavelet transform, an empirical method that allows for estimating the relationship between investment, Q and cash flow with possible variation simultaneously across frequencies and over time.

We find that the information content of Q and cash flow are complementary rather than alternative to each other, since both variable display a valuable but different information content for investment. Q is in fact more important for investment in the medium-to-long run, while cash flow matters more at business cycles frequencies. Moreover, we find that investment-Q sensitivity declines over time at all frequencies, while investment-cash flow sensitivity declines over business cycles frequencies but remains largely stable over the medium-to-long run.

These business cycle findings resembles those obtained using macroeconomic models with the financial accelerator mechanism *à la* Bernanke, Gertler, and Gilchrist (1999), whereas the availability of credit is tied to the firm's leverage, which in turn fluctuates endogenously over the business cycle. In the medium/long run, however, investment is driven by the fundamental value of the firm's projects, thus investment is less sensitive to the cyclical availability of finance. These results also highlight the importance of looking beyond traditional business cycle fluctuations and support recent contributions in the literature (Comin and Gertler 2006; Lubik, Matthes, and Verona 2019) showing that medium/long run cycles are indeed important and should not be neglected by policymakers when conducting stabilization policies. Following the steps of Gallegati, Giri, and Palestrini (2019), an important issue for future research is to bring together these two literatures and study time-frequency decomposition in a general equilibrium model in which investment is explained by the fundamental equity value of the firm in the medium-to-long run, whereas it is more determined by the availability of internal and external finance at business cycle frequencies.

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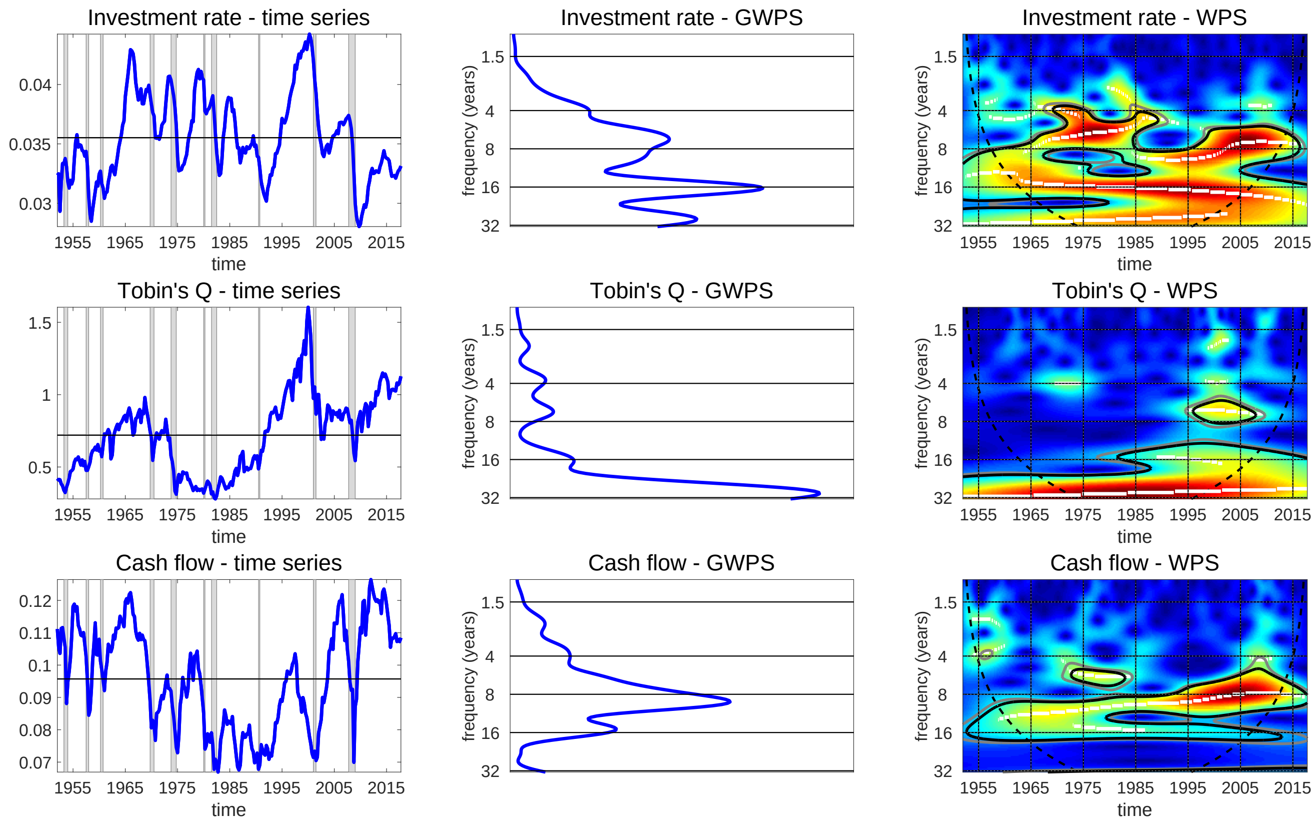

Figure 1: Time series (left column), global wavelet power spectrum (middle column) and wavelet power spectrum (right column) of investment rate (first row), Tobin's Q (middle row) and cash flow (bottom row)

Notes. In the time series plots, gray bars denote NBER recessions and the black horizontal line the time series average of the variable. In the power spectrum charts, the color code ranges from blue (low volatility) to red (high volatility). The black (gray) contour marks significance at the 5% (10%) level and the white stripes mark local maxima. Black dashed lines: cone of influence (indicates the region affected by edge effects).

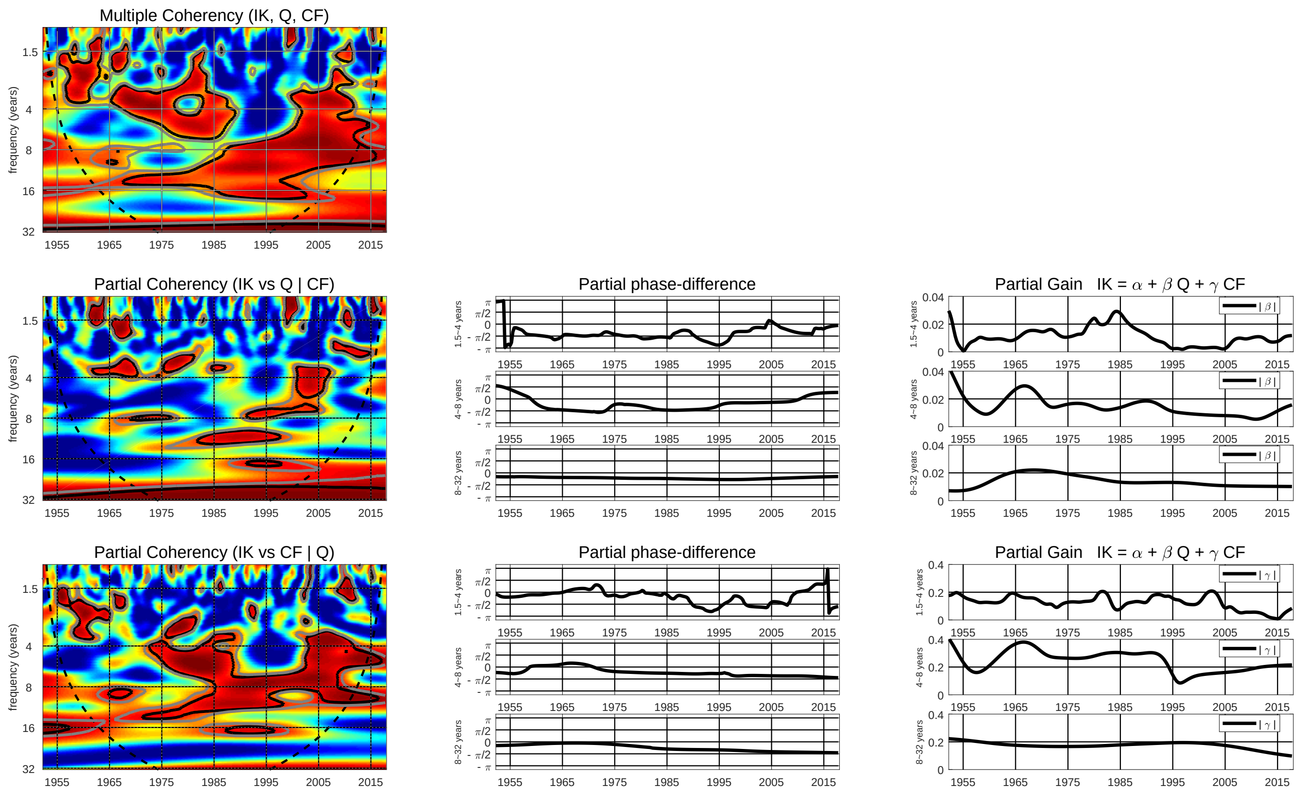

Figure 2: Time-frequency relationship between investment, Tobin's Q and cash flow

Notes. Left column: multiple (top graph) and partial (middle and bottom graphs) wavelet coherencies. The color code ranges from blue (low coherency = close to zero) to red (high coherency = close to one). The black (gray) contour designates the 5% (10%) significance level. Black dashed lines: cone of influence (indicates the region affected by edge effects). Middle column: partial wavelet phase-differences for three frequency intervals. Right column: partial wavelet gains for three different frequency intervals.